Wuhan University
Journal of Natural Sciences

2022, Vol.27 No.2, 153-160

Article ID 1007-1202(2022)02-0153-08
DOI https://doi.org/10.1051/wujns/2022272153

Valuation of Hybrid Pension Scheme Liabilities under Inflation

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Abstract: In recent years, it is of great interest to evaluate the level of liabilities of the hybrid pension system as the mixed pension schemes are favored by various countries around the world. This paper further improves the hybrid pension liability assessment method proposed by Broeders *et al* by accounting for inflation risk and assuming that inflation risk is measured by a price index that follows geometric Brownian motion. A simulation-based pricing framework is then introduced to assess the hybrid pension liability. The results show that the introduction of inflation risk increases the total outstanding liability of hybrid pensions. Furthermore, inflation is negatively correlated with the total outstanding liability of the hybrid pension scheme, while inflation volatility is positively correlated with it.

Key words: hybrid pension schemes; inflation; overlapping generations; stochastic interest rates

CLC number: F 84

Received date: 2021-12-14

Foundation item: Supported by the National Social Science Foundation of China (20BTJ048), Preliminary Research Key Project of National Science Foundation of China (XJKY08201901), and Anhui University Humanities and Social Science Research Major Project (SK2021ZD0043)

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0 Introduction

With the development of private pension systems around the world, the traditional DB(defined benefit) and DC(defined contribution) type pension schemes have all presented different risk characteristics, then hybrid pension schemes that combine the main advantages of both have emerged (Table 1). The characteristic of the DB pension scheme is that the pension scheme sponsor or administrator makes a promise to the scheme participants that their pension benefits will be paid out as agreed in advance. A fixed return is promised to the plan participant with a certain level of security. However, no individual investment accounts are established for plan participants; instead, the plan sponsor invests collectively and manages the plan centrally, and participants are not entitled to excess investment returns. The investment choice and investment risk is borne by the sponsor, and the investment return fail to meet the guaranteed obligations promised to the participants. This represents a long-term funding liability for the sponsor company, which is exposed to solvency risk. The characteristic of the DC pension scheme is that the plan participants contribute a defined contribution to the pension plan, but the participants' benefits upon retirement are determined by the investment return. This is because the plan sponsor establishes individual accounts for plan participants, which are invested by the participants according to their own investment choices and risk appetite, allowing them to earn excess investment returns, but with the investment risk being borne by the participants themselves. However, in the event of major fluctuations in the financial markets, this will result in a shrinkage of the assets in the personal account and the participant's income upon retirement will not be protected. A so-called hybrid pension plan is one in which the plan sponsor promises the plan participant a fixed income at the time of his or her retirement, while establishing an individual account for the plan participant to invest independently. The final benefit at retirement consists of a promised fixed income and an investment income, with not only a fixed guarantee but also an excess investment income. Due to the effective combination of the advantages of DB and DC plans, hybrid pension plans are more complex in terms of design, systems and regulation, and may have conflicting incentives.

Table 1 Comparison of the basic characteristics of DB, DC and hybrid plans

Type of plan	Key benefit	Major disadvantage
DB	Fixed guarantee	Risk is borne by the promoter; Lack of return on excess investments
DC	Self-investment of personal accounts; Excess investment returns	Risk is borne by the participants; Inadequate safeguards
Hybrid pension	Fixed protection; Excess investment income	Complexity in design, systems, regulation, etc.

Some scholars, such as Chen *et al*[1] used the Black-Scholes model to conduct pricing and risk analysis of hybrid pensions combining DB and DC and hybrid pensions with fixed rates of return. Wang[2] reviewed the solvency of the long-term actuarial balance model and application of Social Security in the US and the asset-liability model and application of public pensions in Sweden to provide a reference for the establishment of a solvency assessment model for basic pension insurance in China. Wang *et al*[3] used a continuous-time stochastic model and asset-liability management (ALM) techniques to obtained optimal investment allocations as well as contribution and benefit adjustment strategies with reduced expected losses of plan participants. Fang[4] introduced the main types of hybrid pension schemes and risk-sharing arrangements in the international context, and explores the development prospects of hybrid pension schemes and the implications for the reform of China's annuity system. Khorasanee[5] proposed a hybrid pension scheme and used a stochastic rate of return model to compare the risk sharing and benefit smoothing performance of the hybrid scheme with that of a DC scheme, concluding that the hybrid scheme is more effective in controlling investment risk. Broeders *et al*[6] modelled a continuous hybrid pension plan, evaluating the liabilities of the hybrid pension plan under assumptions such as stochastic interest rates and overlapping generations, and introduced a valuation expression for the liabilities. van Binsbergen *et al*[7] used financial valuation techniques to measure the unfunded PBGC single-employer pension plan- related liabilities by calculating the value of each PBGC- covered plan option liabilities. Devolder *et al*[8] applied Musgrave's rule to the problem of intergenerational risk sharing in a two-cycle framework to obtain an optimal solution within a family of hybrid pension plans. Karim and Jan[9] considered the valuation based on the hedging of mean-variance for the liabilities of mixed pension plans in a multi-cycle dynamic investment environment, and concluded that the level of liabilities was equivalent under fair valuation and mean-variance hedge valuation.

However, as central banks have significantly deflated their currencies, inflation levels have become increasingly high. The investment returns and liability levels of hybrid pension funds have become more and more significantly affected by inflation, leading to a devaluation of the benefits of plan participants and a reduction in the purchasing power of funds. In order to make the valuation of hybrid pension scheme liabilities more accurate and realistic, the inclusion of an inflation factor is necessary and of some relevance. Therefore, based on Broeders *et al*[6], this paper investigated the valuation of liabilities in hybrid pension scheme by taking inflation into account. Firstly, it is assumed that, the hybrid pension assets are invested in two financial assets, equities and bonds, and both obey the corresponding stochastic differential equations, the interest rate obeys the Vasiček stochastic interest rate model, and the inflation risk is measured by the price index obeying the geometric Brownian motion. Secondly, the valuation expressions for the total outstanding liabilities of the hybrid pension scheme are derived from Ito's formula and knowledge of stochastic equation theory. Finally, numerical simulations are conducted to discuss the impact of the mixability parameter, equity allocation ratio, inflation rate, equity volatility and inflation volatility on the valuation of the total outstanding liabilities of the hybrid pension scheme.

1 Model Building

This paper sets up a hybrid pension scheme based on a composition of 55 overlapping generations, assuming that the age of participants in the scheme ranges from 25 to 80 years old, that all individuals start working at 25 years old and retire at 65 years old, that life expectancy is 15 years, that longevity risk is not taken into account and that the 55 groups are homogeneous. The hybrid pension fund starts at $t=0$ and follows a self-financing strategy, with no additional investments or capital transfers from investments over the course of the investment process, other than the initial investment, which is designed to prevent speculation by scheme participants. Over time, the mixed pension fund population in the model will consist of fewer and fewer working generations, until $t=40$, the entire mixed pension fund population consists of retired generations.

1.1 Interest Adjustment Mechanism

Assumption 1 All retired generations initially receive the same benefit at time *t*.

$$Z_i^x = Z_i \tag{1}$$

where Z_i^x denotes the periodic pension benefit paid to the first *x* generation at the time i.

However, benefits are adjusted over time. The plan sponsor is paying pension benefits periodically, and the timing *i* of benefit payments is related to the $[t,i]$, accumulated earnings on the fund's assets and market interest rates, with the benefit adjustment mechanism satisfying

$$Z_i = Z\cdot \mathrm{e}^{y_i - y_t} \tag{2}$$

$$y_u = \alpha r_u^p + (1-\alpha) R_u \tag{3}$$

$$r_u^p = \ln\frac{X_u}{X_0} \tag{4}$$

$$R_u = \int_0^u r_u \mathrm{d}u \tag{5}$$

where Z is a constant. r_u^p is the cumulative log rate of return on pension fund assets *X* over [0, *u*] time. R_u is the cumulative interest rate. The liability is valued at t time. $\alpha\in[0,1]$ is the hybridity parameter. The benefit adjustment mechanism suggests that the final return is a linear combination of the log return on the $r_i^p - r_t^p$ hybrid pension fund assets and the log return on the default-free bonds $R_i - R_t$.

In addition, a continuous pension contract can be defined by the parameters α : when $\alpha=0$, the pension contract has the characteristics of a DB plan because the beneficiary's performance is a risk-free return; when $\alpha=1$, the pension contract has the characteristics of a DC plan because the beneficiary receives a real return on the pension fund assets; when $\alpha\in(0,1)$, the pension contract is a hybrid plan consisting of DB and DC elements.

Substituting equations (3)-(5) into (2) yields

$$Z_i = Z\cdot\left(\frac{X_i}{X_t}\right)^{\alpha}\cdot \mathrm{e}^{\left\{(1-\alpha)\cdot\int_t^i r_u \mathrm{d}u\right\}} \tag{6}$$

1.2 Financial Assets

It is assumed that the hybrid pension fund invested in financial markets contains only two types of traded assets operating during the life of the fund [0, *T*]: one is equities S and the other is risk-free bonds B.

Assumption 2 The evolution of the value of the equity portfolio satisfies the stochastic differential equation.

$$\mathrm{d}S_t = r_t S_t \mathrm{d}t + \sigma_{\mathrm{s}} S_t(\rho \mathrm{d}W_{1,t} + \sqrt{1-\rho^2}\mathrm{d}W_{2,t}) \tag{7}$$

where $W_{1,t}$ and $W_{2,t}$ are two independent one-dimensional Brownian motions and σ_{s} is the volatility of the stock portfolio. The factor ρ denotes the correlation between stock returns and the interest rate market.

Assumption 3 The evolution of the value of bonds satisfies the stochastic differential equation.

$$\mathrm{d}B_t = r_t B_t \mathrm{d}t \tag{8}$$

Inflation risk is measured by a price index that follows a geometric Brownian motion and whose price level Q_t satisfies the differential equation:

$$\mathrm{d}Q_t = Q_t\left(\mu_t \mathrm{d}t + \sigma_t \mathrm{d}\tilde{B}_t\right) \tag{9}$$

where μ_t is the expected inflation rate at the time *t*, $\tilde{B}_t$ denotes the one-dimensional Brownian motion defined on $\{\Omega, F, \{F_t\}_{t\in[0,T]}, P\}$, and σ_t denotes the volatility at the time *t*.

True value of the equity portfolio: $\overline{S}_t = \frac{S_t}{Q_t}$,

$$\begin{aligned}\mathrm{d}\overline{S}_t &= \frac{1}{Q_t}\mathrm{d}S_t + S_t\mathrm{d}\frac{1}{Q_t} + \mathrm{d}S_t\mathrm{d}\frac{1}{Q_t}\\ &= \overline{S}_t\Big[\left(r_t + \sigma_t^2 - \mu_t\right)\mathrm{d}t - \sigma_t\mathrm{d}\tilde{B}_t\\ &\quad +\sigma_{\mathrm{s}}\left(\rho\mathrm{d}W_{1,t} + \sqrt{1-\rho^2}\mathrm{d}W_{2,t}\right)\Big]\end{aligned} \tag{10}$$

True value of bonds: $\overline{B}_t = \frac{B_t}{Q_t}$,

$$\mathrm{d}\overline{B}_t = \frac{1}{Q_t}\mathrm{d}B_t + B_t\mathrm{d}\frac{1}{Q_t} + \mathrm{d}B_t\mathrm{d}\frac{1}{Q_t}$$

$$=\bar{B}_t\left[\left(r_t+\sigma_t^2-\mu_t\right)\mathrm{d}t-\sigma_t\mathrm{d}\tilde{B}_t\right] \tag{11}$$

Assumption 4 Interest rates obey the Vasiček stochastic rate [10] .

$$\mathrm{d}r_t=\left(b-ar_t\right)\mathrm{d}t+\sigma_r\mathrm{d}W_{1,t} \tag{12}$$

where a is the velocity factor, b/a is the long-term average interest rate, and σ_r is the volatility of the short-term interest rate. All the parameters are constants.

In the Vasiček model, under the risk neutral measure p, the cumulative interest rate at time s is expressed as:

$$\begin{aligned}R_s&=\int_0^s r_u\mathrm{d}u\\&=D\left(0,s\right)r_0+b\int_0^s D\left(u,s\right)\mathrm{d}u+\sigma_r\int_0^s D\left(u,s\right)\mathrm{d}W_{1,u}\end{aligned} \tag{13}$$

From equation (13)

$$\begin{aligned}R_s-R_t&=\int_t^s r_u\mathrm{d}u\\&=D\left(t,s\right)r_t+b\int_t^s D\left(u,s\right)\mathrm{d}u+\sigma_r\int_t^s D\left(u,s\right)\mathrm{d}W_{1,u}\end{aligned} \tag{14}$$

of which $D\left(t,s\right)=\mathrm{e}^{at}\cdot\int_t^s\mathrm{e}^{-au}\mathrm{d}u$.

Assumption 5 The hybrid pension fund invests a proportion of its assets in the equity portfolio β and a proportion in risk-free bonds $1-\beta$. The true value of the assets of the hybrid pension fund changes as follows:

$$\begin{aligned}\mathrm{d}X_t&=X_t\left[\beta\frac{\mathrm{d}\bar{S}_t}{\bar{S}_t}+(1-\beta)\frac{\mathrm{d}\bar{B}_t}{\bar{B}_t}\right]\\&=X_t\left[\beta\left(r_t+\sigma_t^2-\mu_t\right)\mathrm{d}t-\beta\sigma_t\mathrm{d}\tilde{B}_t+\beta\sigma_{\mathrm{s}}\left(\rho\mathrm{d}W_{1,t}+\sqrt{1-\rho^2}\mathrm{d}W_{2,t}\right)\right]\\&\quad+(1-\beta)\left(r_t+\sigma_t^2-\mu_t\right)\mathrm{d}t-(1-\beta)\sigma_t\mathrm{d}\tilde{B}_t\\&=X_t\left[\left(r_t+\sigma_t^2-\mu_t\right)\mathrm{d}t-\sigma_t\mathrm{d}\tilde{B}_t+\beta\sigma_{\mathrm{s}}\left(\rho\mathrm{d}W_{1,t}+\sqrt{1-\rho^2}\mathrm{d}W_{2,t}\right)\right]\end{aligned} \tag{15}$$

Given the risk-neutral measure p, applying the Ito formula, the logarithmic return of X_t on [0, t] is expressed as :

$$\begin{aligned}r_t^p&=\ln\left(\frac{X_t}{X_0}\right)\\&=\int_0^t\left(r_t+\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_{\mathrm{s}}^2\right)\mathrm{d}t\\&\quad+\int_0^t\beta\sigma_{\mathrm{s}}\left(\rho\mathrm{d}W_{1,t}+\sqrt{1-\rho^2}\mathrm{d}W_{2,t}\right)-\int_0^t\sigma_t\mathrm{d}\tilde{B}_t\end{aligned} \tag{16}$$

From equation (16),

$$\begin{aligned}r_i^p-r_t^p&=\int_t^i\left(r_t+\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_{\mathrm{s}}^2\right)\mathrm{d}t\\&\quad+\int_t^i\beta\sigma_{\mathrm{s}}\left(\rho\mathrm{d}W_{1,u}+\sqrt{1-\rho^2}\mathrm{d}W_{2,u}\right)-\int_t^i\sigma_t\mathrm{d}\tilde{B}_t\end{aligned} \tag{17}$$

1.3 Total Outstanding Liabilities

The present value of future benefits payable to participants in a hybrid pension plan is considered to be the total outstanding liability of the sponsor of the hybrid pension plan. A distinction is made between two scenarios: $t\le 40$ (both working and retired generations) and $t>40$ (retired generations only). The total outstanding liability is expressed as discounted over time t as follows.

$$L_{t,\mathrm{T}}=\begin{cases}\sum\limits_{x=t}^{40}\left(\sum\limits_{i=41}^{55}\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x-\sum\limits_{i=x}^{40}\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}P_i^x\right)+\sum\limits_{x=41}^{55}\sum\limits_{i=x}^{55}\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x, & t\le 40\\ \sum\limits_{x=t}^{55}\sum\limits_{i=x}^{55}\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x, & t>40\end{cases} \tag{18}$$

where $\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}$ denotes the discount factor and all cash flows are discounted to time t. r_u denotes the stochastic interest rate. In the model, when $t\le 40$, there are both working and retired generations, but working generations pay pension contributions, and the periodic premiums paid P_i^x by working generations x at time i are assumed to be constant. In return, working generations within the hybrid pension plan are entitled to receiving pension benefits in the future. Thus, their target liability is the difference in parentheses in equation (18). Retired generations receive only annual pension benefits.

2 Hybrid Pension Liability Valuation Derivation

The study in this paper is to determine the value of the total outstanding liability t at the time of $t=1,\cdots,55$. The valuation of the liability of a hybrid pension plan at time t can be expressed as:

$$E\left[L_{t,\mathrm{T}}\middle|\mathcal{F}_t\right]=$$

$$\begin{cases}\sum_{x=t}^{40}\left(\sum_{i=41}^{55}E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x\,|\mathcal{F}_t\right]-\sum_{i=x}^{40}E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}P_i^x\,|\mathcal{F}_t\right]\right)\\ \quad+\sum_{x=41}^{55}\sum_{i=x}^{55}E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x\,|\mathcal{F}_t\right], & t\le 40\\ \sum_{x=t}^{55}\sum_{i=x}^{55}E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x\,|\mathcal{F}_t\right], & t>40\end{cases} \tag{19}$$

Since premium payments P_i^x are assumed to be deterministic, the calculation boils down to determining expectations.

$$\bar{Z}_{t,i}=E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i^x\,|\mathcal{F}_t\right] \tag{20}$$

The value of the total outstanding liability $E\left[L_{t,\mathrm{T}}\,|\mathcal{F}_t\right]$ at time t is then determined by replacing this expected value.

Thus, the benefit depends on the cumulative return from t to i and substituting equation (6) into (20) yields

$$\begin{aligned}\bar{Z}_{t,i}&=E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}Z_i\,|\mathcal{F}_t\right]\\ &=Z\cdot E\left[\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}\cdot\mathrm{e}^{\left\{\alpha\left(r_i^p-r_t^p\right)+(1-\alpha)(R_i-R_t)\right\}}\,|\mathcal{F}_t\right]\end{aligned} \tag{21}$$

From equation (14).

$$R_i-R_t=\int_t^i r_u\mathrm{d}u \tag{22}$$

Substituting equations (17) and (22) into equation (21) yields

$$\begin{aligned}&\bar{Z}_{t,i}=Z\\ &\cdot E\left[\mathrm{e}^{\alpha\int_t^i\left(\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_\mathrm{s}^2\right)\mathrm{d}u-\alpha\int_t^i\sigma_t\mathrm{d}\tilde{B}_u+\alpha\int_t^i\beta\sigma_\mathrm{s}\left(\rho\mathrm{d}W_{1,u}+\sqrt{1-\rho^2}\mathrm{d}W_{2,u}\right)}\,|\mathcal{F}_t\right]\\ &=Z\\ &\cdot\mathrm{e}^{\alpha\left(\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_\mathrm{s}^2\right)(i-t)}E\left(\mathrm{e}^{\alpha\int_t^i\beta\sigma_\mathrm{s}\left(\rho\mathrm{d}W_{1,u}+\sqrt{1-\rho^2}\mathrm{d}W_{2,u}\right)-\alpha\int_t^i\sigma_t\mathrm{d}\tilde{B}_u}\right)\end{aligned} \tag{23}$$

Among them, the cumulative interest rate from t to i in the income accumulation coefficient offsets the discount factor. The solution in the expectation of $\alpha\int_t^i\beta\sigma_\mathrm{s}(\rho\mathrm{d}W_{1,u}+\sqrt{1-\rho^2}\mathrm{d}W_{2,u})-\alpha\int_t^i\sigma_t\mathrm{d}\tilde{B}_u$ of equation (23) is a normal distribution that obeys an expectation of 0 and a variance of $\alpha^2(\sigma_t^2+\sigma_\mathrm{s}^2\beta^2)$ $(i-t)$ (Note that $\alpha\int_t^i\beta\sigma_\mathrm{s}(\rho\mathrm{d}W_{1,u}+\sqrt{1-\rho^2}\mathrm{d}W_{2,u})-\alpha\int_t^i\sigma_t\mathrm{d}\tilde{B}_u$ is an expression for the variable upper bound function on i).

Solving equation (23) yields

$$\bar{Z}_{t,i}=Z\cdot\mathrm{e}^{\left[\alpha^2\left(\frac{1}{2}\beta^2\sigma_\mathrm{s}^2+\frac{1}{2}\sigma_t^2\right)+\alpha\left(\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_\mathrm{s}^2\right)\right](i-t)} \tag{24}$$

In summary, substituting equation (24) into equation (19) yields

$$E\left[L_{t,\mathrm{T}}\,|\mathcal{F}_t\right]=$$

$$\begin{cases}\sum_{x=t}^{40}\left(\sum_{i=41}^{55}Z\cdot\mathrm{e}^{\left[\alpha^2\left(\frac{1}{2}\beta^2\sigma_\mathrm{s}^2+\frac{1}{2}\sigma_t^2\right)+\alpha\left(\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_\mathrm{s}^2\right)\right](i-t)}-\sum_{i=x}^{40}\mathrm{e}^{-\int_t^i r_u\mathrm{d}u}P_i^x\right)\\ \quad+\sum_{x=41}^{55}\sum_{i=x}^{55}Z\cdot\mathrm{e}^{\left[\alpha^2\left(\frac{1}{2}\beta^2\sigma_\mathrm{s}^2+\frac{1}{2}\sigma_t^2\right)+\alpha\left(\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_\mathrm{s}^2\right)\right](i-t)}, & t\le 40\\ \sum_{x=t}^{55}\sum_{i=x}^{55}Z\cdot\mathrm{e}^{\left[\alpha^2\left(\frac{1}{2}\beta^2\sigma_\mathrm{s}^2+\frac{1}{2}\sigma_t^2\right)+\alpha\left(\frac{1}{2}\sigma_t^2-\mu_t-\frac{1}{2}\beta^2\sigma_\mathrm{s}^2\right)\right](i-t)}, & t>40\end{cases} \tag{25}$$

3 Numerical Analysis and Insights

3.1 Data Sources

For calculation purposes, the initial liability, premium amount, is assumed to be in unit 1. The parameters related to inflation are sourced from the data of Huo *et al*[11] and the parameters of the stochastic interest rate model are sourced from the data of Brennan and Xia[12] (see Table 2). The long-run mean of interest rates is 0.0315/0.63 = 0.05. The initial interest rate level is assumed to be equal to the long-run mean level of interest rates to offset the effect of systematic increases or decreases in interest rates.

Table 2 Relevant parameters sourced from references

Variable	Value	Symbolic
Phase	55	T
Initial liability	1	Z
Premium amount	1	P_i^x
Initial interest rate	0.05	r_0
Interest rate volatility	0.026	σ_r
Speed factor	0.63	a
Other parameter	0.013 5	b
Stock volatility	0.25	σ_s
Correlation coefficient	−0.129	ρ
Equity allocation ratio	0.5	β
Mixability parameter	0.5	α
Inflation rate	0.028-0.032	μ_t
Inflation volatility	0.3	σ_t

3.2 Numerical Simulation

Based on the theoretical valuation expression introduced in equation (25), the parameters are brought into the calculation and numerical simulations are performed using Matlab software to analyze and compare the impact of the mixability parameter, equity allocation ratio, and stock volatility on total outstanding liabilities (L), as well as the impact of inflation rate and inflation volatility on them, with and without considering the inflation risk, respectively.

The numerical simulation results are shown in Figs. 1-4.

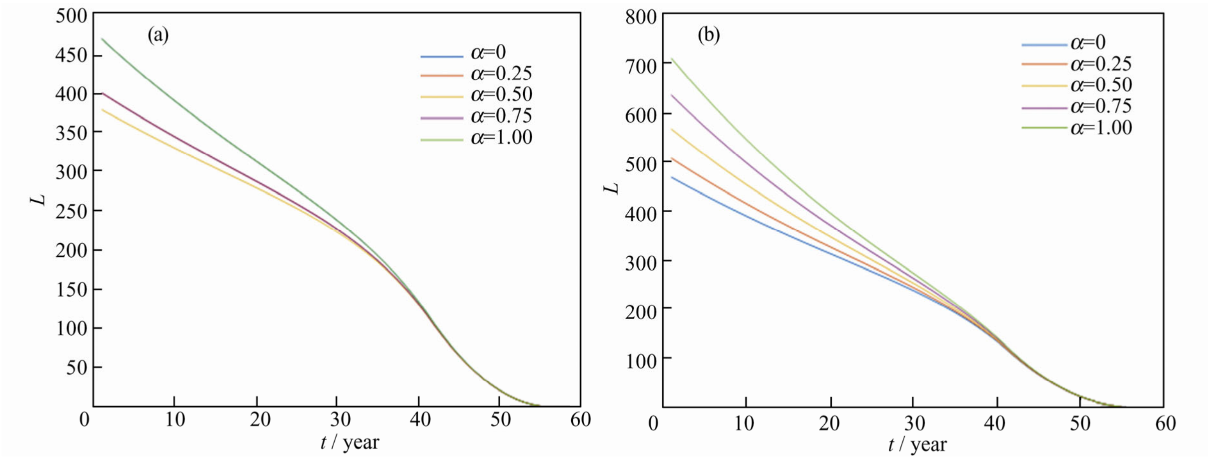

Fig. 1 Effect of the mixability parameter on total outstanding liabilities

(a) without considering the inflation risk; (b) with considering the inflation risk

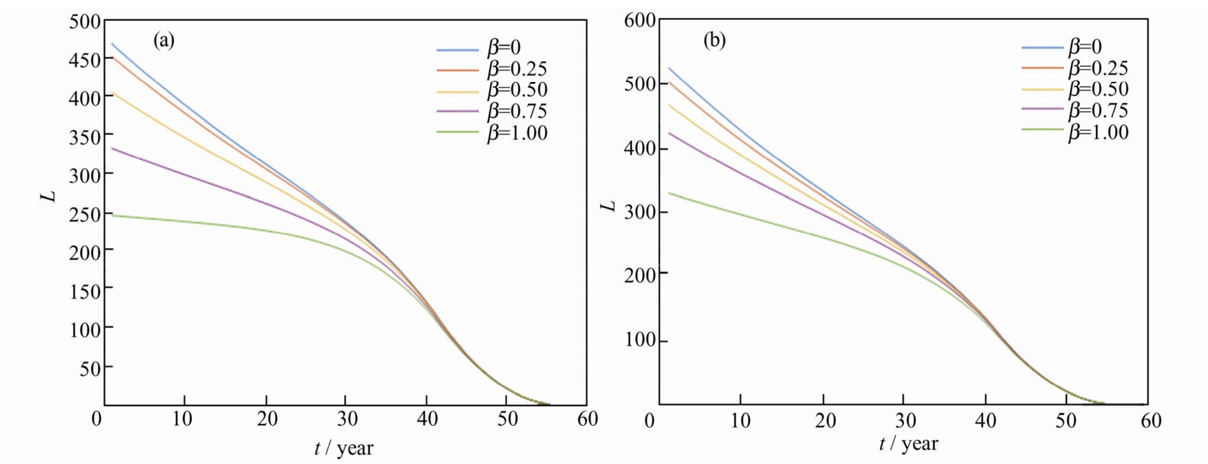

Fig. 2 Effect of equity allocation ratio on total outstanding liabilities

(a) without considering the inflation risk; (b) with considering the inflation risk

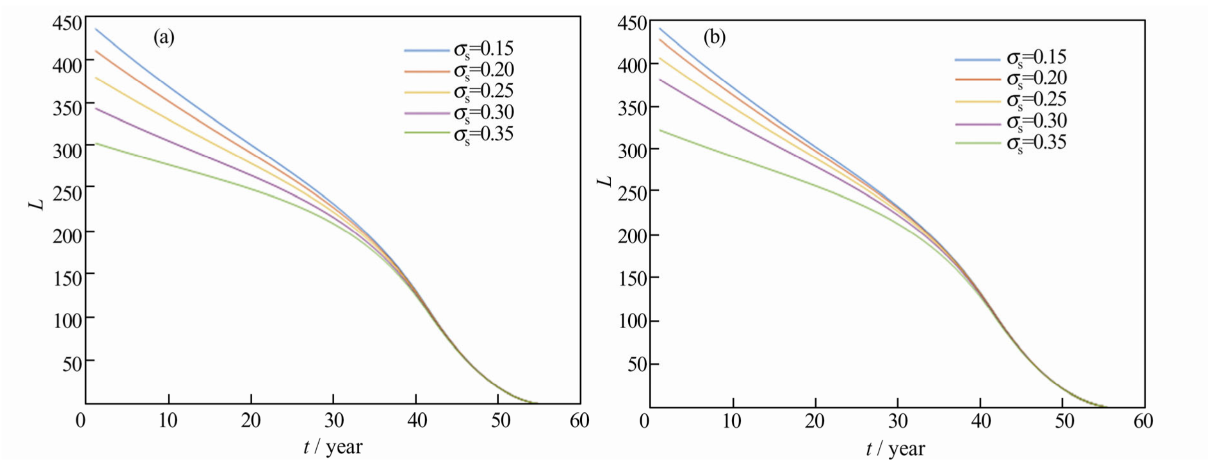

Fig. 3 Effect of stock volatility on total outstanding liabilities

(a) without considering the inflation risk; (b) with considering the inflation risk

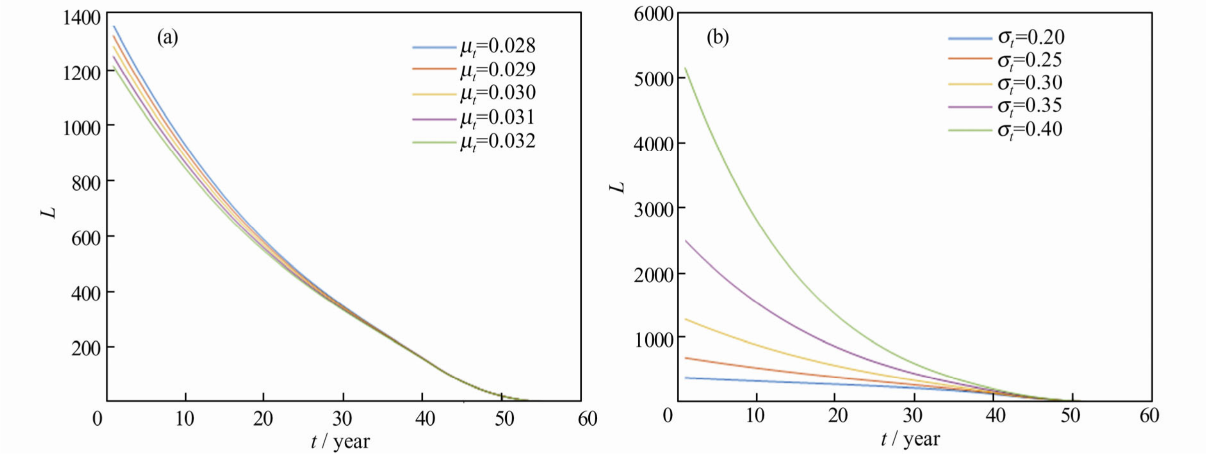

Fig. 4 Effect of inflation rate (a) and inflation volatility (b) on total outstanding liabilities

3.3 Discussion and Insights

From Fig. 1, total outstanding liabilities without inflation risk are a symmetric function with respect to the mixing parameter, achieving a minimum at $\alpha = 0.5$. Total outstanding liabilities under inflation risk, on the other hand, are positively correlated with the mixability parameter, due to the fact that the inclusion of inflation risk changes the structure of the original valuation formula. The value of total outstanding liabilities under consideration of inflation risk is higher than that under non-inflation risk for the same mixability parameter. This is because the plan sponsor is required to bear a portion of the inflation risk of the DB, resulting in an increase in benefits paid to plan participants. The degree that the mixability parameter affects the total outstanding liability of the hybrid pension plan diminishes over time. Because with the passage of time, the maturity date of hybrid pension funds is shortened, resulting in the shortening of the fluctuation cycle of asset return.

From Fig. 2, the equity allocation ratio is negatively related to the total outstanding liability of the hybrid pension. The investment accounts in the DC-type portion of the hybrid plan are invested at the individual's own expense, and the larger the equity allocation ratio, the smaller the defined benefit assumed by the plan sponsor. For the same equity allocation ratio, the value of the total outstanding liability with inflation risk taken into account is higher than that without inflation risk. As the equity allocation ratio increases, the magnitude of the decline in the total outstanding liability with inflation risk is smaller than the decrease in total outstanding liabilities without considering inflation risk. This is due to the presence of inflation risk, and the increased benefit payments by plan sponsors offset a portion of the decline in total outstanding liabilities due to the equity allocation ratio.

From Fig. 3, the stock volatility is negatively correlated with the total outstanding liability of hybrid pensions. Under given conditions, an increase in stock volatility implies that investment risk exposure increases and is borne by the plan participants themselves. For the same stock volatility, the value of the total outstanding liability with inflation risk taken into account is higher than the value of the total outstanding liability without inflation risk. As stock volatility increases, the rate of decline in the total outstanding liability with inflation risk is higher than the rate of decline in the total outstanding liability without inflation risk as inflation risk increases. This is caused by the superposition of inflation risk and exposure to equities.

From Fig. 4(a), there is a negative correlation between the inflation rate and the total outstanding liability of hybrid pensions. A gradual increase in the inflation rate and a decrease in the purchasing power of funds leads to a decrease in the benefits actually received by plan participants and hence a decrease in the total outstanding liability of the plan sponsor. The inflation rate affects the benefits of the hybrid pension plan to a greater extent at the beginning of the period, but gradually becomes weaker as time goes on. From Fig. 4(b), there is a positive relationship between inflation volatility and the total outstanding liability of hybrid pensions. The impact of different inflation volatility rates on the total outstanding liability varies considerably at the beginning of the period, but the extent of the impact fades as the period of fund operation decreases. As inflation volatility gradually increases, the total outstanding liability increases to an increasing extent.

The implication of the above findings is that, for plan participants, hybrid pension plans expand residents' pension strategy choices, but plan participants should choose the mixability parameters and equity investment ratios rationally, taking into account their own asset positions, investment preferences, and investment horizons. For the government: 1) A multi-level and multi-channel pension system should be established through active and steady implementation of mixed pension plan reforms; 2) Reasonable mixed pension parameters and equity investment ratios should be formulated. This is because in hybrid pension funds, plan sponsors prefer a higher proportion of mixability parameters and equity investment ratios, which can reduce the benefits paid to plan participants on a regular basis. Conversely, plan participants prefer lower blending parameters and equity investment ratios, which can increase their own benefit receipt. Therefore, the government should establish a reasonable range of mixability parameters and equity investment ratios to protect both sides of the benefits of a hybrid pension plan. For the financial market: 1) Multiple channels should be developed to finance plan sponsors' funds and explore the issuance of inflation-indexed bonds and inflation-indexed funds actively to hedge against inflation risks; 2) Plan sponsors can use reinsurance and with-profits annuity business in the financial market to transfer part of the inflation risk; 3) The investment scope of pension funds should be expand to change the situation of low investment returns, single investment method and restricted investment types for pensions, and it has become particularly important to realize the value preservation and appreciation of pensions, etc.

4 Summary and Outlook

In this paper, we consider that inflation obeys a stochastic differential equation and that taking inflation into account when calculating hybrid pension assets will make the calculated valuation of hybrid pension liabilities more realistic. The evolution of hybrid pension assets under inflation risk is first modeled. Then, knowledge of stochastic differential equations is applied to derive an expression for the valuation of the hybrid pension liability under inflation risk. Finally, the numerical analysis of the relevant parameters shows that the introduction of inflation risk increases the total outstanding liabilities of hybrid pensions. In addition, equity allocation ratio, equity volatility and inflation are negatively related to the total outstanding liabilities of hybrid pension plans, while the mixability parameter and inflation volatility are positively related to them. The study draws relevant insights to the design of policy or hybrid pension plans.

Although the model for valuing the total outstanding liability of hybrid pensions incorporates inflation risk as a systematic risk, it still has some limitations. The next step is to consider longevity risk, which also has an impact on the valuation of the total outstanding liabilities of hybrid pensions, and its study is also of some relevance.

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