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Correspondence

Exploring the Power of Entangled Data in Quantum Machine Learning

□ **WANG Xinbiao[1,2,3], DU Yuxuan[3,4], TU Zhuozhuo[5], LUO Yong[1,2†], YUAN Xiao[6,7], TAO Dacheng[4]**

1. Institute of Artificial Intelligence, School of Computer Science, Wuhan University, Wuhan 430072, Hubei, China;
2. National Engineering Research Center for Multimedia Software, Wuhan University, Wuhan 430072, Hubei, China;
3. JD Explore Academy, Beijing 101111, China;
4. School of Computer Science and Engineering, Nanyang Technological University, Singapore 639798, Singapore;
5. School of Computer Science, Faculty of Engineering, University of Sydney, NSW 2008, Australia;
6. Center on Frontiers of Computing Studies, Peking University, Beijing 100871, China;
7. School of Computer Science, Peking University, Beijing 100871, China

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Quantum entanglement is a key resource for achieving superiority of quantum computing. Currently, scientists are extensively focusing on how to integrate quantum entanglement into various components of quantum machine learning (QML) models, aiming to surpass the performance of traditional machine learning models. Notable successes include the use of entangled measurements[1-3] and entangled channels[4], which have been shown to reduce query complexity or improve the prediction precision for specified QML tasks. Quantum entangled data, capable of encoding more information compared to classical data of the same size, is recognized for its potential to achieve quantum advantages. Nevertheless, the impact of the entanglement degree in quantum data on model performance remains a challenging and unresolved research question.

Recently, Wang and collaborators[5] (*Nat Commun* 2024, 15, 3716) rigorously analyzed the impact of the entanglement degree of quantum data, the number and type of measurements, and the size of training dataset on the prediction error of QML models by establishing the quantum no-free-lunch (NFL) theorem. They consider tasks of quantum dynamics learning with entangled data, which is fundamental to many advanced quantum algorithms, as illustrated in Fig. 1. While conventional understanding suggests that entanglement mainly confers benefits to QML in terms of sample complexity[6,7], this study demonstrates that the effect of the degree of entanglement on prediction error exhibits a dual effect, that is, whether quantum entanglement improves performance depends on the number of measurements allowed. With a sufficient number of measurements, increasing the entanglement degree in quan-

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Biography: WANG Xinbiao, Ph. D. candidate, research direction: quantum machine learning algorithms and theory, and quantum information theory. E-mail: wangxb08@whu.edu.cn

† Corresponding author. E-mail: luoyong@whu.edu.cn

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tum data can effectively reduce the prediction error, or decrease the size of the training dataset needed to achieve the same level of prediction error. Conversely, using highly entangled data may increase the prediction error when only a few measurements are permitted.

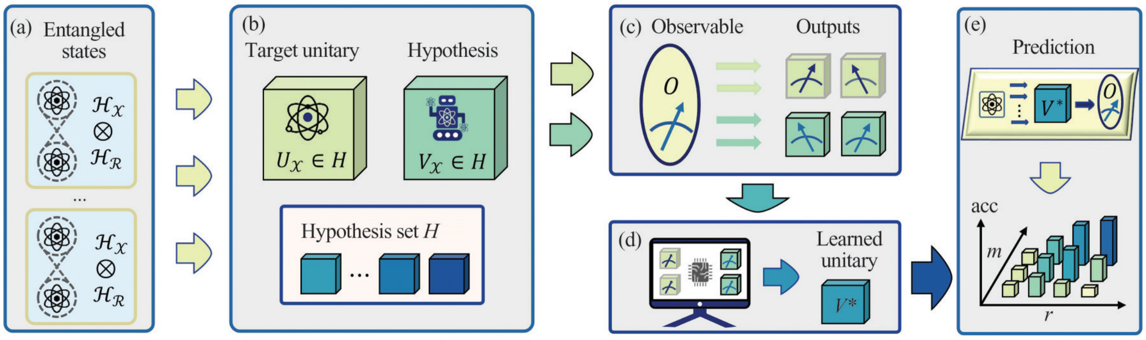

Fig.1 Illustration of quantum NFL setting with the entangled data[5]

The goal of the quantum learner is to learn a unitary $V_{\mathcal{X}}$ that can accurately predict the output of the target unitary $U_{\mathcal{X}}$ under a fixed observable O, where the subscript $\mathcal{X}$ refers to the quantum system in which the operator O act on. The learning process is as follows. (a) A total number of N entangled bipartite quantum states living in Hilbert space $\mathcal{H}_{\mathcal{X}}\otimes\mathcal{H}_{\mathcal{R}}$ ($\mathcal{R}$ denotes the reference system) are taken as inputs, dubbed entangled data. (b) Quantum learner proceeds incoherent learning. The entangled data separately interacts with the target unitary $U_{\mathcal{X}}$ (agnostic) and the candidate hypothesis $V_{\mathcal{X}}$ extracted from the same Hypothesis set H. (c) The quantum learner is restricted to leverage the finite measured outcomes of the observable O on the output states of $U_{\mathcal{X}}$ and $V_{\mathcal{X}}$ to conduct learning. (d) A classical computer is exploited to infer V^* that best estimates $U_{\mathcal{X}}$ according to the measurement outcomes. For example, in the case of variational quantum algorithms, the classical computer serves as an optimizer to update the tunable parameters of the ansatz $V_{\mathcal{X}}$. (e) The learned unitary V^* is used to predict the output of unseen quantum states in Hilbert space $\mathcal{H}_{\mathcal{X}}$ under the evolution of the target unitary $U_{\mathcal{X}}$ and the measurement of O. A large Schmidt rank r can enhance the prediction accuracy when combined with a large number of measurements m, but may lead to a decrease in accuracy when m is small.

This research provides practical guidance for designing advanced quantum learning protocols, especially those tailored for current quantum computers with limited quantum computing resources.

On the theoretical side, the results of Wang and collaborators are an important reminder of the entangled data utilization, suggesting that extracting classical information from entangled data is difficult although they can store more information than unentangled data. An important research direction motivated by this study is inquiring whether there exists a similar transition role when exploiting entangled measurements which has more powerful information-extraction capabilities from entangled data and has been shown to achieve an exponential advantage under many specific learning tasks[1-3].

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