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The Space-Time Semi-Analytical Meshless Methods for Coupled Burgers' Equations

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Abstract: In this paper, a simple direct space-time semi-analytical meshless scheme is proposed for the numerical approximation of the coupled Burgers' equations. During the whole solution procedure, two different schemes are considered in terms of radial and non-radial basis functions. The time-dependent variable in the first radial scheme is directly considered as the normal space variables to formulate an "isotropic" space-time radial basis function. The second non-radial scheme considered relationship between time-dependent and space-dependent variables. Under such circumstance, we can get a one-step space-time meshless scheme. The numerical findings demonstrate that the proposed meshless schemes are precise, user-friendly, and effective in solving the coupled Burgers' equations.

Key words: radial basis functions; coupled Burgers' equations; meshless methods; numerical simulation

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0 Introduction

The coupled Burgers' equation is a fundamental partial differential equation with applications in mathematical physics[1]. For two-dimensional cases, it has the form

$$\begin{cases} \dfrac{\partial U}{\partial t} + U\dfrac{\partial V}{\partial x} + V\dfrac{\partial V}{\partial y} - \dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2}\right) = 0, \\ \dfrac{\partial V}{\partial t} + U\dfrac{\partial U}{\partial x} + V\dfrac{\partial U}{\partial y} - \dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 U}{\partial y^2}\right) = 0, \end{cases} \quad (x,y) \in \Omega \tag{1}$$

where Re is a real constant known as the Reynolds number.

For such time-dependent problems, it is very difficult to get the corresponding theoretical/exact solutions. Several numerical methods have been proposed and discussed for solving the coupled Burgers' equation[2, 3]. Almost all these numerical methods are based on the finite-difference-method[4] or similarity transform[5].

The radial basis function (RBF) -based meshless methods[6-8], which abandon the mesh generation in Finite Element Method (FEM), have fascinated many scholars' attention. Siraj-ul-Islam *et al*[9] used a local RBFs collocation method to get the approximate solution of the nonlinear coupled Burgers' equations. Based on RBFs, Ahmad *et al*[10] investigated a new local meshless method for the numerical simulation of 1D Klein-Gordon and 2D coupled Burgers' equations[11]. Two sys-

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tems of integrable coupled Burgers-type equations are discussed by Wazwaz[12]. Jaradat[13] investigated multiple kink solutions and other exact solutions for a two-mode coupled Burgers' equation. Based on the FEM, a Galerkin quadratic B-spline FEM is proposed for a coupled Burgers' equation[14]. For the coupled viscous Burgers' equation with very large values of the Reynolds number, solutions are investigated by Başhan[15]. It should be pointed that the above-mentioned numerical method are all two-step methods, i.e., the finite difference method is used to discretize the time variable and then another method can be used to find numerical solutions for time-independent problems.

In this paper, we propose a space-time semi-analytical meshless method, which is a one-step method, for the two-dimensional coupled Burgers' equations. Two different strategies are proposed for the RBFs and non-RBFs. The time variable in the first radial scheme is treated equally as space variables which yields an "isotropic" space-time radial basis function. A relationship that is grounded in reality between space variables and time variable is investigated by the non-radial scheme. Under such circumstances, the time variable and space variables can be treated simultaneously during the whole solution process and two-dimensional coupled Burgers' equations can be solved in a direct way.

The rest of the paper is as follows. In Section 1, we describe the two different schemes for the RBFs and non-RBFs. Followed by Section 2, the methodology of the space-time semi-analytical meshless method (SSMM) is proposed for the two-dimensional coupled Burgers' equations under initial condition and boundary conditions. Two numerical examples are presented to validate the accuracy and stability of the proposed algorithms in Section 3. Conclusion is given in Section 4 with some future directions.

1 The Space-Time RBFs and Non-RBFs

To get the solution of the 2D coupled Burgers' equation, initial and boundary conditions should be considered simultaneously as

$$\begin{cases} U(x,y,0)=g_1(x,y), \quad V(x,y,0)=g_2(x,y), \quad (x,y)\in\Omega \\ U(x,y,t)=g_3(x,y,t), V(x,y,t)=g_4(x,y,t), \quad (x,y)\in\partial\Omega \end{cases} \tag{2}$$

Traditional numerical methods typically employ two-level finite difference approximations or integral transform methods to solve Eq. (1) with the initial and boundary conditions. To overcome the limitations of the two-level strategy, we suggest utilizing direct meshless methods that employ both space-time radial and non-radial basis functions.

It is widely recognized that radial basis functions exhibit an "isotropic" behavior in Euclidean spaces. For steady-state problems, the approximate solution can be written as a linear combination of RBFs with 2D or more higher dimensions. Take the famous Multiquadric (MQ) RBF as an example

$$\phi_{\mathrm{MQ}}(r_j)=\sqrt{1+(\varepsilon r_j)^2}, \tag{3}$$

where $r_j=\|X-X_j\|$ is the Euclidean distance between two points $X=(x,y)$ and $X_j=(x_j,y_j)$, ε is the RBF shape parameter.

However, there is only one space variable x for the 2D coupled Burgers' equation, the traditional RBFs are unapplicable in the direct sense. For this reason, we propose a simple meshless method by combining the space variable x and time variable t from the perspective of radial and non-radial.

More precisely, the interval $[a,b]$ is divided into equal segments $a=x_0<x_1<\cdots<x_n=b$, and the corresponding fineness is denoted as $h=(b-a)/n$. The time variable is uniformly selected from the initial time $t_0=0$ to a final time $t_n=T$, i. e., $0=t_0<t_1<\cdots<t_n=T$ with a time-step of $\Delta t=T/n$. Figure 1 depicts the configuration of the space-time coordinate. Then the space-time RBF can be constructed as

$$\varphi_{\mathrm{MQ}}(r_j)=\sqrt{1+c^2 r_j^2}, \tag{4}$$

where $r_j=\|P-P_j\|=\sqrt{(x-x_j)^2+(t-t_j)^2}$. Besides, we can construct the space-time non-RBF which can be expressed as

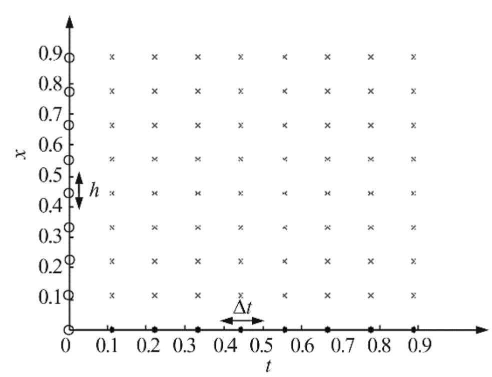

Fig. 1 Configuration of the space-time coordinate

"○" represents the value of the space variable x, "•" denotes the value of the time variable t, and "×" indicates the point (x,t)

$$\varphi_{\mathrm{NMQ}}(P,P_j)=\sqrt{1+(x-x_j)^2+c^2(t-t_j)^2}, \tag{5}$$

where c reflects a relationship that is grounded in reality between x and t.

In Ref. [16], an investigation is done on the non-radial basis function in space-time, which is the product of two positive definite functions, one on the space dimension and the other on the time dimension. For the MQ case, one has

$$\varphi'_{\mathrm{NMQ}}(P,P_j)=\sqrt{1+c^2(x-x_j)^2}\sqrt{1+c^2(t-t_j)^2}. \tag{6}$$

Nevertheless, the numerical findings do not effectively address the issues in this study. Obtaining the space-time radial and non-radial basis functions is a straightforward task for two-dimensional cases

$$\begin{cases}\varphi_{\mathrm{MQ}}(r_j)=\sqrt{1+c^2r_j^2}\\ \varphi_{\mathrm{NMQ}}(P,P_j)=\sqrt{1+(x-x_j)^2+(y-y_j)^2+c^2(t-t_j)^2}\end{cases} \tag{7}$$

with $r_j=\|P-P_j\|$ is the Euclidean distance between $P=(x,y,t)$ and $P_j=(x_j,y_j,t_j)$.

2 Implementation of the Space-Time Semi-Analytical Meshless Method

In this section, we examine the direct meshless method (SSMM) by considering the initial boundary value problem expressed in Eqs. (1) and (2). By utilizing the space-time radial and non-radial basis functions, it is possible to solve Eqs. (1) and (2) in a one level approximation directly. The approximate solution can be presented as

$$\bar{u}(\cdot)\approx\sum_{j=1}^{N}\lambda_j\varphi_j(\cdot), \tag{8}$$

with $\{\lambda_j\}_{j=1}^{n}$ the unknown coefficients.

In order to demonstrate the SSMM, we select collocation points across the entire physical domain which include internal points N_{I}, initial boundary points N_{t} and boundary points N_{b}. According to the traditional collocation method, by substituting Eq. (8) into Eqs. (1)-(2), we obtain

$$\sum_{j=1}^{N}\lambda_j L_1\varphi_j(P_i,P_j)=0,\ i=1,\cdots,N_{\mathrm{I}}, \tag{9}$$

$$\sum_{j=1}^{N}\lambda_j L_2\varphi_j(P_i,P_j)=0,\ i=1,\cdots,N_{\mathrm{I}}, \tag{10}$$

$$\sum_{j=1}^{N}\lambda_j\varphi_j(P_i,P_j)=g_1(P_i),\ \ i=N_{\mathrm{I}}+1,\cdots,N_{\mathrm{I}}+N_{\mathrm{t}}, \tag{11}$$

$$\sum_{j=1}^{N}\lambda_j\varphi_j(P_i,P_j)=g_2(P_i),\ \ i=N_{\mathrm{I}}+1,\ldots,N_{\mathrm{I}}+N_{\mathrm{t}}, \tag{12}$$

$$\sum_{j=1}^{N}\lambda_j\varphi_j(P_i,P_j)=g_3(P_i),\ \ i=N_{\mathrm{I}}+N_{\mathrm{t}}+1,\ldots,N, \tag{13}$$

$$\sum_{j=1}^{N}\lambda_j\varphi_j(P_i,P_j)=g_4(P_i),\ \ i=N_{\mathrm{I}}+N_{\mathrm{t}}+1,\ldots,N, \tag{14}$$

where $L_1\varphi_j=L_2\varphi_j=\dfrac{\partial\varphi_j}{\partial t}+\varphi_j\dfrac{\partial\varphi_j}{\partial x}+\varphi_j\dfrac{\partial\varphi_j}{\partial y}-\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2\varphi_j}{\partial x^2}+\dfrac{\partial^2\varphi_j}{\partial y^2}\right)$. Obviously, the total number of collocation points is $N=N_{\mathrm{I}}+N_{\mathrm{t}}+N_{\mathrm{b}}$.

Consequently, we should find out the solution of the following systems

$$\boldsymbol{A}_1\boldsymbol{X}_1=\boldsymbol{f}_1, \tag{15}$$

$$\boldsymbol{A}_2\boldsymbol{X}_2=\boldsymbol{f}_2, \tag{16}$$

where

$$\boldsymbol{A}_1=\begin{bmatrix}\boldsymbol{A}_{11}&\boldsymbol{A}_{12}&\boldsymbol{A}_{13}\\ \boldsymbol{A}_{21}&\boldsymbol{A}_{22}&\boldsymbol{A}_{23}\\ \boldsymbol{A}_{31}&\boldsymbol{A}_{32}&\boldsymbol{A}_{33}\end{bmatrix} \tag{17}$$

are $N\times N$ known matrix with submatrices

$$\boldsymbol{A}_{11}=\{L_1\varphi_j(P_i,P_j)\}_{j=1}^{N_{\mathrm{I}}},\ \boldsymbol{A}_{12}=\{L_1\varphi_j(P_i,P_j)\}_{j=N_{\mathrm{I}}+1}^{N_{\mathrm{I}}+N_{\mathrm{t}}},$$
$$\boldsymbol{A}_{13}=\{L_1\varphi_j(P_i,P_j)\}_{j=N_{\mathrm{I}}+N_{\mathrm{t}}+1}^{N},$$

for $i=1,2,\ldots,N_{\mathrm{I}}$,

$$\boldsymbol{A}_{21}=\{\varphi_j(P_i,P_j)\}_{j=1}^{N_{\mathrm{I}}},\ \boldsymbol{A}_{22}=\{\varphi_j(P_i,P_j)\}_{j=N_{\mathrm{I}}+1}^{N_{\mathrm{I}}+N_{\mathrm{t}}},$$
$$\boldsymbol{A}_{23}=\{\varphi_j(P_i,P_j)\}_{j=N_{\mathrm{I}}+N_{\mathrm{t}}+1}^{N},$$

for $i=N_{\mathrm{I}}+1,\cdots,N_{\mathrm{I}}+N_{\mathrm{t}}$,

$$\boldsymbol{A}_{31}=\{\varphi_j(P_i,P_j)\}_{j=1}^{N_{\mathrm{I}}},\ \boldsymbol{A}_{32}=\{\varphi_j(P_i,P_j)\}_{j=N_{\mathrm{I}}+1}^{N_{\mathrm{I}}+N_{\mathrm{t}}},$$
$$\boldsymbol{A}_{33}=\{\varphi_j(P_i,P_j)\}_{j=N_{\mathrm{I}}+N_{\mathrm{t}}+1}^{N},$$

for $i=N_{\mathrm{I}}+N_{\mathrm{t}}+1,\cdots,N$.

$$\boldsymbol{X}_1=\begin{bmatrix}\lambda^1\\ \lambda^2\\ \lambda^3\end{bmatrix} \tag{18}$$

is $N\times 1$ vectors.

$$\boldsymbol{f}_1=\begin{bmatrix}f_1\\ f_2\\ f_3\end{bmatrix} \tag{19}$$

is $N\times 1$ vectors with $\boldsymbol{f}_1=[0,0,\cdots,0]^{\mathrm{T}}$,

$$\boldsymbol{f}_2=\left[g_1\left(P_{N_{\mathrm{I}}+1}\right),g_1\left(P_{N_{\mathrm{I}}+2}\right),\cdots,g_1\left(P_{N_{\mathrm{I}}+N_{\mathrm{t}}}\right)\right]^{\mathrm{T}},$$
$$\boldsymbol{f}_3=\left[g_3\left(P_{N_{\mathrm{I}}+N_{\mathrm{t}}+1}\right),g_3\left(P_{N_{\mathrm{I}}+N_{\mathrm{t}}+2}\right),\cdots,g_3\left(P_N\right)\right]^{\mathrm{T}}.$$

The similar expressions can be easily obtained for $\boldsymbol{A}_2$, $\boldsymbol{X}_2$ and $\boldsymbol{f}_2$. Equations (15) and (16) can be solved by the backslash computation in MATLAB codes. From the above procedures, we can find that the implementation of the proposed SSMM is very simple.

3 Numerical Experiments

To compare with the previous literatures, we consider using the maximum error. Our current research does not cover the optimal choice of RBF parameter. For more information on this topic, readers can refer to Ref.[17] and references therein. The shape parameter for the 2D coupled Burgers' equations is chosen by prior numerical results. For simplicity, we denote the space-time RBF Eq. (3) and space-time non-RBF Eq. (4) as SSMM1 and SSMM2, respectively.

3.1 Case 1

We consider the 2D Burgers' equations, with the exact solutions

$$\begin{cases} U(x,y,t)=\dfrac{3}{4}-\dfrac{1}{4\left[1+\exp(-4x+4y-t)(\mathrm{Re}/32)\right]}, \\ V(x,y,t)=\dfrac{3}{4}+\dfrac{1}{4\left[1+\exp(-4x+4y-t)(\mathrm{Re}/32)\right]}. \end{cases} \tag{19}$$

The equation above specifies the initial condition at time t. The boundary conditions, which are also determined by the equation above, vary as time t changes. Here, the Reynolds number is $\mathrm{Re}=80$, and the uniform mesh grid $h_x=h_y=1/8$ is chosen for the SSMM, while the uniform mesh grid $h_x=h_y=0.05$ is used in Ref. [18] for all problems but the time t is different[18].

The absolute errors for numerical solutions and exact solutions are given in Table 1 with $t=0.05$, 0.2, 0.5 and different locations. From Table 1 we can see that the SSMM performs better than the discrete ADM in Ref. [18]. Note that the proposed method under the first scheme SSMM1 has similar results with the method under the second scheme SSMM2.

For the uniform mesh grid $h_x=h_y=\dfrac{1}{8}$ and shape parameter $c=1$, we have provided the corresponding figures for numerical solutions and exact solutions in Figs. 2 and 3. To show the solution accuracy at different t, we consider solution for U at $t=0.1$ and solution for V at $t=0.5$.

3.2 Case 2

For this case, we consider the 2D Burgers' equations on computational domain $D=\{(x,y)|0\leqslant x,t\leqslant 0.5\}$ with the exact solutions

$$\begin{cases} U(x,y,t)=\dfrac{x+y-2xt}{1-2t^2}, \\ V(x,y,t)=\dfrac{x-y-2yt}{1-2t^2}. \end{cases} \tag{20}$$

Table 1 The absolute errors for numerical solutions for *U* and *V* at different *t*

t	Test point	U-SSMM1	U [18]	V-SSMM1	V [18]
0.05	(0.1,0.1)	3.0E−06	1.3E−04	9.1E−06	1.3E−04
	(0.8,0.3)	1.2E−05	1.0E−05	2.6E−05	1.0E−05
	(0.7,0.4)	6.1E−05	6.0E−05	6.0E−05	6.0E−05
	(0.9,0.5)	1.0E−05	3.0E−05	7.4E−06	3.0E−05
	(0.1,0.6)	2.0E−04	1.0E−05	6.0E−05	1.0E−05
	(0.8,0.6)	3.1E−06	9.0E−05	6.8E−06	9.0E−05
	(0.3,0.7)	2.1E−04	1.0E−05	1.1E−05	1.0E−05
	(0.9,0.9)	3.5E−06	1.3E−04	1.3E−05	1.3E−04
0.2	(0.1,0.1)	1.7E−04	2.7E−04	9.1E−06	2.6E−05
	(0.8,0.3)	2.9E−05	4.0E−05	2.6E−05	4.2E−05
	(0.7,0.4)	1.9E−04	1.9E−04	6.0E−05	8.5E−04
	(0.9,0.5)	1.3E−04	9.0E−05	7.4E−06	3.9E−05
	(0.1,0.6)	9.5E−04	2.0E−05	6.0E−05	3.0E−05
	(0.8,0.6)	5.5E−05	2.9E−04	6.8E−06	8.2E−06
	(0.3,0.7)	2.3E−05	3.0E−05	1.1E−05	2.6E−05
	(0.9,0.9)	1.5E−04	2.7E−04	2.7E−04	6.5E−04
0.5	(0.1,0.1)	1.5E−04	3.4E−04	4.0E−04	3.3E−04
	(0.8,0.3)	2.7E−04	1.9E−04	1.1E−04	1.9E−04
	(0.7,0.4)	1.5E−04	7.5E−04	1.2E−05	7.4E−04
	(0.9,0.5)	1.6E−04	4.4E−04	4.5E−06	4.4E−04
	(0.1,0.6)	7.6E−05	9.0E−05	6.9E−05	9.0E−05
	(0.8,0.6)	2.4E−04	1.1E−04	2.2E−03	1.1E−04
	(0.3,0.7)	6.3E−05	2.9E−04	6.0E−05	2.9E−04
	(0.9,0.9)	5.0E−07	3.4E−04	5.9E−05	3.3E−04

The initial conditions are $U(x,y,0)=x+y$ and $V(x,y,0)=x-y$.

Numerical results of the presented SSMM are compared with the discrete ADM in Ref. [18]. At $t=0.1$ and $t=0.4$, the uniform mesh is chosen as $h_x=h_y=\dfrac{1}{8}$ and $h_x=h_y=\dfrac{1}{9}$, respectively. It should be mentioned that the uniform mesh $h_x=h_y=0.025$ used in Ref. [18], i. e., the points used in Ref. [18] are much more than those in the present SSMM.

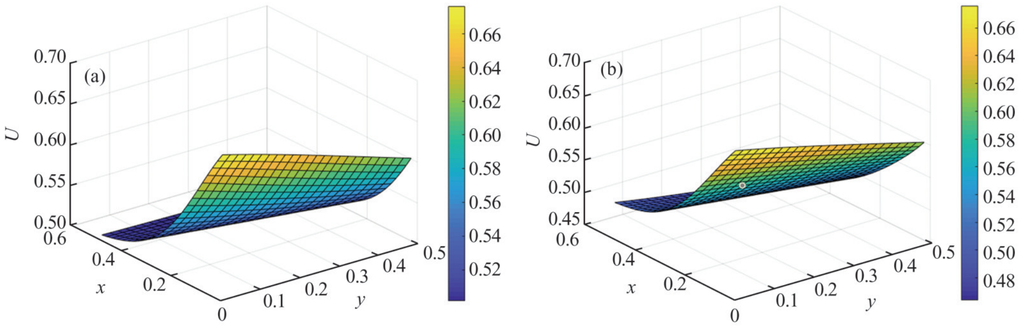

Fig. 2 Exact solution U (a) and numerical solution U (b) at time $t=0.1$ with uniform mesh grid $h_x=h_y=\frac{1}{8}$ and shape parameter $c=1$

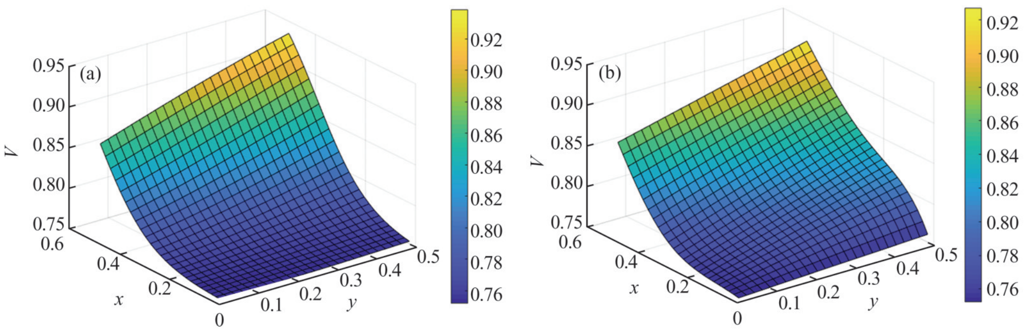

Fig. 3 Exact solution V (a) and numerical solution V (b) at time $t=0.5$ with uniform mesh grid $h_x=h_y=\frac{1}{8}$ and shape parameter $c=1$

For $t=0.1$ and $t=0.4$, numerical solutions with Reynolds number $\mathrm{Re}=1$ are listed in Table 2. It can be seen from Table 2 that the present SSMM performs better than the discrete ADM at $t=0.1$ for test point $(x,y)=(0.5,0.5)$. While the numerical results for the other test points are almost the same. Numerical results in Table 2 show that the approximation solutions by SSMM perform better than the discrete ADM for all test points. Therefor one can conclude that the SSMM is an accurate and efficient method to solve a nonlinear system of equations. From practical opinions, the numerical results may reduce with the increase of time t. The SSMM is more stable than the discrete ADM with the increase of time t.

For the uniform mesh grid $h_x=h_y=0.025$ and shape parameter $c=0.4$, we have provided the corresponding figures for numerical solutions and exact solutions at time $t=0.1$ in Figs. 4 and 5.

Table 2 The absolute errors for numerical solutions for U and V at different t

t	Test Point	U-SSMM1	U [18]	V-SSMM1	V [18]	U-SSMM2	V-SSMM2
	(0.1,0.1)	3.98E−05	3.31E−06	3.16E−05	1.05E−06	9.24E−06	1.47E−05
	(0.2,0.2)	3.93E−05	6.62E−06	9.04E−05	2.11E−06	3.85E−05	1.69E−05
0.1	(0.3,0.3)	2.98E−05	9.92E−06	2.82E−05	3.16E−06	3.05E−04	2.39E−05
	(0.5,0.5)	8.78E−06	1.65E−05	5.75E−06	5.27E−06	4.95E−06	2.52E−06
	(0.1,0.1)	8.29E−05	1.02E−04	3.90E−05	3.55E−04	1.58E−05	3.99E−05
	(0.2,0.2)	5.19E−04	2.04E−04	2.15E−04	7.10E−04	1.65E−04	5.64E−06
0.4	(0.3,0.3)	4.05E−04	3.06E−04	7.13E−05	1.06E−03	2.11E−05	2.21E−05
	(0.5,0.5)	3.52E−06	5.10E−04	6.69E−06	1.77E−03	2.07E−04	9.60E−06

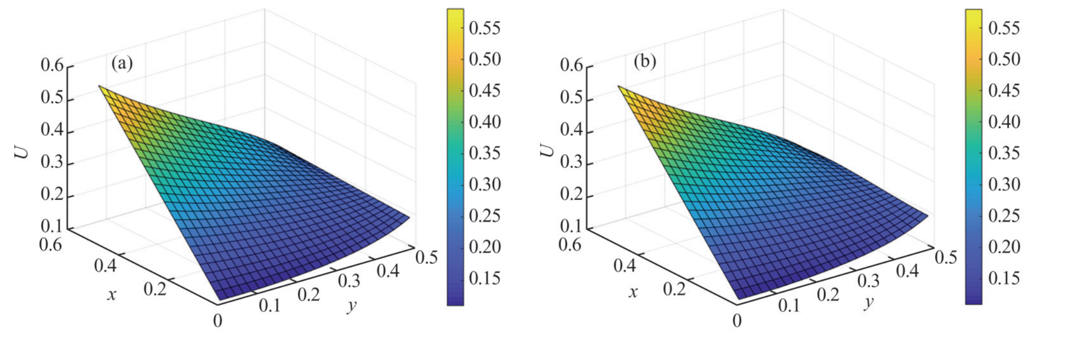

Fig. 4 Exact solution U (a) and numerical solution U (b) at time $t=0.1$ with uniform mesh grid $h_x=h_y=0.025$ and shape parameter $c=0.4$

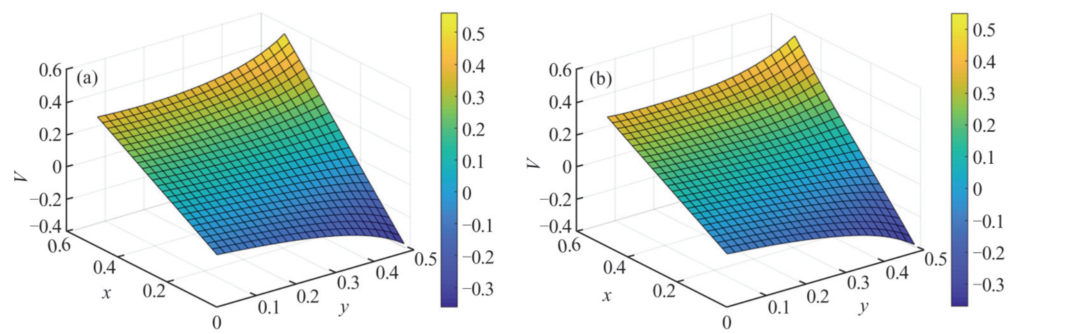

Fig. 5 Exact solution V (a) and numerical solution V (b) at time $t=0.1$ with uniform mesh grid $h_x=h_y=0.025$ and shape parameter $c=0.4$

4 Conclusion

In this paper, a new space-time semi-analytical meshless method is proposed for the 2D coupled Burgers' equations. For the basic functions from radial and non-radial aspects, two approaches are suggested. The first strategy is realized by building an "isotropic" space-time radial basis function by treating the time variable as a normal space variable. The alternative plan took into account a practical, non-radial link between the space and time variables. For the Klein-Gordon equations, both of the suggested meshless method's schemes are straightforward, precise, reliable, simple to program, and effective. What's more, the suggested approach works with iteration techniques for nonlinear situations. Our SSMM procedure's theory can be immediately applied to high-dimensional thermo-elastic issues, transient heat transfer, and wave propagation.

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耦合Burgers方程的时空半解析无网格方法

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摘要：本文提出了一种简单的半解析直接时空无网格格式用于耦合Burgers方程的数值近似求解。在求解过程中，考虑了径向基函数和非径向基函数两种不同的方案。第一种径向基函数方案中的时间相关变量被直接视为正常空间变量，进而给出了“各向同性”时空径向基函数；第二种非径向基函数方案考虑了时间相关和空间相关变量之间的关系。在这种情况下，我们可以得到一种一步式时空无网格方案。数值结果表明，所提出的无网格方法精度良好、使用友好，并能有效地求解耦合的Burgers方程。

关键字：径向基函数；耦合Burgers方程；无网格方法；数值模拟

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