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Modified Fixed-Time Synchronization Criteria of Complex Networks with Time-Varying Delays via Continuous or Discontinuous Control

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Abstract: This paper investigates modified fixed-time synchronization (FxTS) of complex networks (CNs) with time-varying delays based on continuous and discontinuous controllers. First, for the sake of making the settling time (ST) of FxTS is independent of the initial values and parameters of the CNs, a modified fixed-time (FxT) stability theorem is proposed, where the ST is determined by an arbitrary positive number given in advance. Then, continuous controller and discontinuous controller are designed to realize the modified FxTS target of CNs. In addition, based on the designed controllers, CNs can achieve synchronization at any given time, or even earlier. And control strategies effectively solve the problem of ST related to the parameters of CNs. Finally, an appropriate simulation example is conducted to examine the effectiveness of the designed control strategies.

Key words: complex networks; settling time; fixed-time synchronization; controllers; time-varying delays

CLC number: TP13

0 Introduction

Complex networks (CNs) have attracted increasing attention in many fields, for examples, social networks^[1], electrical power networks^[2], biological networks and aviation networks, etc^[3-4]. Synchronization is an important issue in CNs and has been extensively studied in recent years, such as asymptotic synchronization^[5-6], and exponential synchronization^[7-8]. Asymptotic synchronization of CNs with structure uncertainty is solved in Ref. [9]. In Ref. [10], exponential synchronization is realized by proposing a sampled data controller for complex dy-

namical networks. However, asymptotic and exponential synchronization are achieved when time tends to infinity, which leads a lot of wasted resources in practical applications.

In order to achieve synchronization of CNs as fast as possible, researchers have proposed finite-time synchronization (FnTS) strategy, and FnTS of various neural networks has been well-studied^[11-13]. Refs. [14] and [15] studied FnTS of dynamical networks with several weights, which provides a better description of actual networks. FnTS of memristive dynamical networks and finite-time (FnT) stability of singular dynamical net-

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works with time delays are considered in Refs.[16] and [17], respectively. Notably, the settling time (ST) of FnTS depends on the initial values of the considered neural networks. However, since these initial values are generally unknown or even unavailable in practical networks, the exact ST remains theoretically indeterminable.

Therefore, for the purpose of obtaining the ST when the initial values of the neural networks are unknown, the strategy of fixed-time synchronization (FxTS) is introduced by Polyakov^[18]. And since the ST of the FxTS is independent of the initial values, FxTS strategy has a wider range of applications than FnTS. In recent years, FxTS has been widely used in communication security^[19-20], bioengineering^[21-22], and financial transaction^[23]. FxTS of coupled neural networks was studied in Ref.[24], where the case of dynamical networks containing discontinuous activation and mismatched parameter is considered. Ref.[25] discussed FxTS of drive and response networks with noise disturbances and discontinuous nodes, and this model can well simulate the workings of neurons and solve some bioengineering problems. Fixed-time (FxT) group consensus on dynamical networks with multiple nodes was studied in Ref. [26], and the implementation of FxT group consensus can greatly increase the security of communication. In these papers and Ref.[27], the ST of FxTS are all determined by the disturbed parameters regardless of the initial values of the neural networks. However, the parameters of the neural networks are usually uncertain due to the presence of some perturbations. Therefore, it is a challenge to achieve FxTS, where the ST is independent of the initial values and disturbed parameters.

In addition, as a tool for achieving FxTS, continuous and discontinuous controllers are often used. Refs. [28] and [29] explored FxT synchronous behaviour of CNs by continuous controllers, which can effectively avoid chattering during the synchronization process. Ref. [30] addressed the FxTS of coupled CNs with delays under the discontinuous controllers, where the controllers contain the symbolic functions.

Through the above statements, it is easy to find that FxTS of CNs without time delays has been fully investigated in Refs.[20-27, 29]. But as we all know, time delays, especially time-varying delays, are unavoidable in engineering applications because of the delayed response of the neural networks and the limited speed of signal propagation. Numerous studies have investigated

the synchronization of CNs with time delays^[31-32]. In Refs. [33] and [34], global exponential synchronization and asymptotic exponential synchronization of CNs with time-delay were investigated by designing controllers, respectively. In Ref. [35], FxTS of CNs with time-varying delays was achieved, where the ST was determined by the turbulent parameters of the considered networks and the controller was complicated. Moreover, few studies have investigated FxTS in CNs with time-varying delays whose ST remains unaffected by network parameters.

This paper aims to explore FxTS of CNs with time-varying delays via continuous or discontinuous control. The key contributions of this paper are given below: (1) Suitable continuous and discontinuous controllers are constructed for implementing FxTS of CNs, and the controllers given in this paper may be simpler than those in the existing literatures^[27-30, 35]. (2) In contrast to the existing result^[35], this paper achieves true FxTS. In other words, the ST is independent of the initial values and the parameters of the considered networks.

The remainder of this paper is arranged as follows. Section 1 presents the model of CNs and preliminaries. Some control strategies are provided for FxTS of CNs in Section 2. A simulation example is formulated in Section 3. Finally, Section 4 gives the conclusion.

1 Model Description and Preliminaries

Let $\mathbb{R}$ and $\mathbb{R}^+$ represent the sets of real numbers and nonnegative real numbers, respectively. $\mathbb{R}^n$ means the n dimensional real space equipped with the Euclidean norm $\|\cdot\|$. $\lambda_{\max}(\mathbf{A})$ is the maximum eigenvalue of matrix $\mathbf{A}$. The notations $\text{sign}(\cdot)$ denotes sign function. $\otimes$ means the Kronecker product.

Consider a class of CNs with time-varying delays whose model is formulated as:

$$\begin{aligned} \dot{\mathbf{r}}_i(t) = & \mathbf{f}(\mathbf{r}_i(t)) + \mathbf{g}(\mathbf{r}_i(t - \delta(t))) + \sum_{j=1}^N m_{ij} \mathbf{A} \mathbf{r}_j(t) \\ & + \sum_{j=1}^N p_{ij} \mathbf{A} \mathbf{r}_j(t - \delta(t)) \end{aligned} \quad (1)$$

with initial condition

$$\mathbf{r}_i(s) = \mathbf{Y}_i(s), \quad s \in [-\delta, 0],$$

where $\mathbf{r}_i(t) = (\mathbf{r}_{i1}(t), \mathbf{r}_{i2}(t), \dots, \mathbf{r}_{in}(t))^T \in \mathbb{R}^n$ denotes neuron state vector, $i = 1, 2, \dots, N$. $\mathbf{f}(\mathbf{r}_i(t)) = (\mathbf{f}_1(\mathbf{r}_{i1}(t)), \mathbf{f}_2(\mathbf{r}_{i2}(t)), \dots, \mathbf{f}_n(\mathbf{r}_{in}(t)))^T$ and $\mathbf{g}(\mathbf{r}_i(t)) = (\mathbf{g}_1(\mathbf{r}_{i1}(t)), \mathbf{g}_2(\mathbf{r}_{i2}(t)), \dots, \mathbf{g}_n(\mathbf{r}_{in}(t)))^T$

are nonlinear vector functions. $\delta(t)$ is the time-varying delays and meets $0 \leq \delta(t) \leq \delta$. A is the internal coupling matrix. $M=(m_{ij})_{N \times N}$ and $P=(p_{ij})_{N \times N}$ are the coupling configuration matrices. Suppose there is a connection between nodes i and j ($i \neq j$), then m_{ij} (or p_{ij}) > 0 , otherwise, m_{ij} (or p_{ij}) $= 0$, and the diagonal elements of matrix M and P are defined by

$$m_{ii} = - \sum_{j=1, j \neq i}^N m_{ij}, \quad 1 \leq i \leq N,$$

$$p_{ii} = - \sum_{j=1, j \neq i}^N p_{ij}, \quad 1 \leq i \leq N.$$

The controlled CNs corresponding to (1) can be described as below:

$$\begin{aligned} \dot{\mathbf{w}}_i(t) = & \mathbf{f}(\mathbf{w}_i(t)) + \mathbf{g}(\mathbf{w}_i(t - \delta(t))) + \sum_{j=1}^N m_{ij} A \mathbf{w}_j(t) \\ & + \sum_{j=1}^N p_{ij} A \mathbf{w}_j(t - \delta(t)) + \mathbf{x}_i(t) \end{aligned} \quad (2)$$

with initial condition

$$\mathbf{w}_i(s) = \Psi_i(s), \quad s \in [-\delta, 0],$$

where $\mathbf{w}_i(t) = (\mathbf{w}_{i1}(t), \mathbf{w}_{i2}(t), \dots, \mathbf{w}_{in}(t))^T \in \mathbb{R}^n$ is the state vector and $\mathbf{x}_i(t) = (\mathbf{x}_{i1}(t), \mathbf{x}_{i2}(t), \dots, \mathbf{x}_{in}(t))^T \in \mathbb{R}^n$ denotes the appropriate controller.

Define an error vector $\kappa_i(t) = \mathbf{w}_i(t) - \mathbf{r}_i(t)$, $i = 1, 2, \dots, N$. Then the synchronization error neural networks can be expressed as

$$\begin{aligned} \dot{\kappa}_i(t) = & \mathbf{F}(\kappa_i(t)) + \mathbf{G}(\kappa_i(t - \delta(t))) + \sum_{j=1}^N m_{ij} A \kappa_j(t) \\ & + \sum_{j=1}^N p_{ij} A \kappa_j(t - \delta(t)) + \mathbf{x}_i(t) \end{aligned} \quad (3)$$

with initial condition

$$\kappa_i(s) = \Psi_i(s) - \Upsilon_i(s), \quad s \in [-\delta, 0],$$

where

$$\mathbf{F}(\kappa_i(t)) = \mathbf{f}(\mathbf{w}_i(t)) - \mathbf{f}(\mathbf{r}_i(t)),$$

$$\mathbf{G}(\kappa_i(t - \delta(t))) = \mathbf{g}(\mathbf{w}_i(t - \delta(t))) - \mathbf{g}(\mathbf{r}_i(t - \delta(t))).$$

Assumption 1 For $\forall \mathbf{r}_i(t), \mathbf{w}_i(t) \in \mathbb{R}^n$, there are positive constants L_1 and L_2 such that

$$|\mathbf{f}(\mathbf{w}_i(t)) - \mathbf{f}(\mathbf{r}_i(t))| \leq L_1 |\mathbf{w}_i(t) - \mathbf{r}_i(t)|,$$

$$|\mathbf{g}(\mathbf{w}_i(t)) - \mathbf{g}(\mathbf{r}_i(t))| \leq L_2 |\mathbf{w}_i(t) - \mathbf{r}_i(t)|.$$

Definition 1 (FnT Stability^[24]) For a class of neural network

$$\dot{\mathbf{z}}(t) = \mathbf{f}(t, \mathbf{z}, \beta), \quad \mathbf{z}(0) = \mathbf{z}_0, \quad (4)$$

where β is the neural network parameter.

The trivial solution $\mathbf{z} = 0$ of neural network (4) is said to be globally FnT stable, if it satisfies globally stable and $\mathbf{z}(t, \mathbf{z}_0) = 0$ for $\forall t \geq T(\mathbf{z}_0)$, where $T: \mathbb{R}^n \rightarrow \mathbb{R}^+ \cup \{0\}$ is called ST function.

Definition 2 (FxT Stability^[28]) For neural network (4), the trivial solution $\mathbf{z} = 0$ is globally FxT stable, if it meets globally FnT stable and $\exists T_{\max} > 0$, subject to the ST function $T(\mathbf{z}_0) \leq T_{\max}$ for $\forall \mathbf{z}_0 \in \mathbb{R}^n$.

Remark 1 In Definition 1, neural network (4) achieves FnT stabilization at $T(\mathbf{z}_0)$, and the ST function $T(\mathbf{z}_0)$ is connected to the initial value $\mathbf{z}_0$ of (4). In Definition 2, neural network (4) achieves FxT stabilization at $T_{\max}$, and $T_{\max}$ is associated with the parameters β of neural network (4).

Definition 3 (Modified FxT Stability^[35]) For neural network (4), the trivial solution $\mathbf{z} = 0$ is called modified FxT stable, if it meets globally FnT stable and for $\forall T^\Delta > 0$ (T^Δ is arbitrarily given positive number in advance), subject to the ST function $T(\mathbf{z}_0) \leq T^\Delta$ for $\forall \mathbf{z}_0 \in \mathbb{R}^n$.

Remark 2 It is clear that Definition 3 is not the same as Definition 1 and Definition 2. In Definition 3, T^Δ is an arbitrary positive value given in advance, which is unrelated to the initial value and the parameter of neural network (4). Thus, the ST in Definition 3 is also irrelevant to the initial values and parameters.

Lemma 1^[29] Given $\phi_i \geq 0$ for $i = 1, 2, \dots, n$, $0 < \eta \leq 1$, and $\zeta > 1$, then

$$\sum_{i=1}^n \phi_i^\eta \geq \left(\sum_{i=1}^n \phi_i \right)^\eta, \quad \sum_{i=1}^n \phi_i^\zeta \geq n^{1-\zeta} \left(\sum_{i=1}^n \phi_i \right)^\zeta.$$

Lemma 2^[36] Suppose there exists the Lyapunov function V such that

$$\dot{V}(\mathbf{z}(t)) \leq -\lambda V^\zeta(\mathbf{z}(t)), \quad t \geq t_0, \quad V(\mathbf{z}(t_0)) \geq 0,$$

where $\lambda > 0$, $0 < \zeta < 1$. Then the trivial solution of neural network (4) is globally FnT stable, and for any given t_0 , $V(\mathbf{z}(t))$ fulfils the inequality as follows:

$$V^{1-\zeta}(\mathbf{z}(t)) \leq V^{1-\zeta}(\mathbf{z}(t_0)) - \lambda(1-\zeta)(t-t_0), \quad t_0 \leq t \leq t_*,$$

and $V(\mathbf{z}(t)) = 0$, $t \geq t_*$, where the ST function t_* is given below

$$t_* = t_0 + \frac{V^{1-\zeta}(\mathbf{z}(t_0))}{\lambda(1-\zeta)}.$$

Lemma 3 Suppose there exists the Lyapunov function V such that

$$\dot{V}(\mathbf{z}(t)) \leq -\frac{1}{T^\Delta} (\mu_1 V^{c_1}(\mathbf{z}(t)) + \mu_2 V^{c_2}(\mathbf{z}(t))), \quad (5)$$

then the trivial solution of neural network (4) is modified globally FxT stable, that is $V(\mathbf{z}(t)) = 0$, $t \geq T$, and T satisfies the inequality as following $T \leq T^\Delta$, where $\mu_1 = (2/(c_1 - 1))$, $\mu_2 = (2/(1 - c_2))$, $c_1 > 1$, $0 < c_2 < 1$, and $T^\Delta > 0$ is a predetermined arbitrary time.

Proof Define the function $H(V) = V^2$, then

$$\dot{H}(V) = 2V\dot{V} \leq -2V \frac{1}{T^\Delta} (\mu_1 V^{c_1} + \mu_2 V^{c_2}) = -2 \frac{1}{T^\Delta} (\mu_1 V^{c_1-c_2} + \mu_2) V^{1+c_2} = -2 \frac{1}{T^\Delta} (\mu_1 H^{\frac{c_1-c_2}{2}} + \mu_2) H^{\frac{1+c_2}{2}}.$$

As $c_1 > 1$, $0 < c_2 < 1$, then $\mu_1 H^{((c_1-c_2)/2)} > 0$ and $\dot{H}(V) \leq -2(\mu_2/T^\Delta) H^{((1+c_2)/2)}$, and based on Lemma 2, the trivial solution of (4) is globally FnT stable.

Combining (5), we can obtain $\frac{dV}{dt} \leq -\frac{1}{T^\Delta} (\mu_1 V^{c_1} + \mu_2 V^{c_2})$, so

$$dt \leq -\frac{dV}{\frac{1}{T^\Delta} (\mu_1 V^{c_1} + \mu_2 V^{c_2})}.$$

Note that $t \rightarrow T$, $V(z(t)) \rightarrow 0$, one may have

$$T = \int_0^T dt \leq \int_{V(z(0))}^0 \frac{dV}{-\frac{1}{T^\Delta} (\mu_1 V^{c_1} + \mu_2 V^{c_2})} = \int_0^{V(z(0))} \frac{dV}{\frac{1}{T^\Delta} V^{c_2} (\mu_1 V^{c_1-c_2} + \mu_2)} = \frac{1}{1-c_2} \int_0^{V(z(0))} \frac{dV^{1-c_2}}{\frac{1}{T^\Delta} (\mu_1 V^{c_1-c_2} + \mu_2)}.$$

Define $\alpha = V^{1-c_2}$, then

$$T \leq \frac{1}{1-c_2} \int_0^{(V(z(0)))^{1-c_2}} \frac{d\alpha}{\frac{1}{T^\Delta} (\mu_1 \alpha^{\frac{c_1-c_2}{1-c_2}} + \mu_2)}.$$

Without loss of generality, we consider two possible cases as follows:

(i) When $(V(z(0)))^{1-c_2} \leq 1$,

$$T \leq \frac{1}{1-c_2} \int_0^{(V(z(0)))^{1-c_2}} \frac{d\alpha}{\frac{1}{T^\Delta} (\mu_1 \alpha^{\frac{c_1-c_2}{1-c_2}} + \mu_2)} \leq \frac{1}{1-c_2} \int_0^1 \frac{d\alpha}{\frac{1}{T^\Delta} (\mu_1 \alpha^{\frac{c_1-c_2}{1-c_2}} + \mu_2)} \leq \frac{1}{1-c_2} \frac{T^\Delta}{\mu_2} = \frac{T^\Delta}{2}.$$

(ii) When $(V(z(0)))^{1-c_2} > 1$,

$$\begin{aligned} T &\leq \frac{1}{1-c_2} \int_0^{(V(z(0)))^{1-c_2}} \frac{d\alpha}{\frac{1}{T^\Delta} (\mu_1 \alpha^{\frac{c_1-c_2}{1-c_2}} + \mu_2)} = \frac{1}{1-c_2} \int_0^1 \frac{d\alpha}{\frac{1}{T^\Delta} (\mu_1 \alpha^{\frac{c_1-c_2}{1-c_2}} + \mu_2)} + \frac{1}{1-c_2} \int_1^{(V(z(0)))^{1-c_2}} \frac{d\alpha}{\frac{1}{T^\Delta} (\mu_1 \alpha^{\frac{c_1-c_2}{1-c_2}} + \mu_2)} \\ &\leq \frac{1}{1-c_2} \int_0^1 \frac{d\alpha}{\frac{\mu_2}{T^\Delta}} + \frac{1}{1-c_2} \int_1^{(V(z(0)))^{1-c_2}} \frac{d\alpha}{\frac{\mu_1}{T^\Delta} \alpha^{\frac{c_1-c_2}{1-c_2}}}. \end{aligned}$$

By definition of α , we can deduce that $\alpha^{\frac{c_1-c_2}{1-c_2}} > 1$, $\int_1^{(V(z(0)))^{1-c_2}} \frac{d\alpha}{\alpha^{\frac{c_1-c_2}{1-c_2}}} < \frac{1-c_2}{c_1-1}$, so $T \leq \frac{T^\Delta}{\mu_2} \frac{1}{1-c_2} + \frac{T^\Delta}{\mu_1} \frac{1}{1-c_2} \frac{1-c_2}{c_1-1} = T^\Delta$.

From (i) and (ii), we have $T \leq T^\Delta$. The proof is completed.

Remark 3 In Lemma 3, if there exist positive numbers $c_1 > 1$ and $0 < c_2 < 1$ such that $\dot{V}(z(t)) \leq -\frac{\mu}{T^\Delta} (V^{c_1}(z(t)) + V^{c_2}(z(t)))$, $\mu = \max\{\mu_1, \mu_2\}$ (or $\mu = \mu_1 + \mu_2$). Then, the trivial solution of (4) is globally FxT stable, and $T \leq T^\Delta$.

2 Theoretical Results

In this section, the strategies of continuous and discontinuous control are provided for achieving the modified FxTS of CNs (1) and (2), respectively.

2.1 Modified FxTS with Continuous Controller

Theorem 1 Under Assumption 1, the continuous controller is

$$x_i(t) = -d_1 \kappa_i(t) - d_2 \kappa_i(t - \delta(t)) - \frac{I}{T^\Delta} (\mu_3 \kappa_i^{b_1}(t) + \mu_4 \kappa_i^{b_2}(t)), \quad (6)$$

then CNs (1) and (2) can achieve modified FxTS, and the ST is $T \leq T^\Delta$, where $d_1 \geq L_1 + \lambda_{\max}\{\mathbf{M} \otimes \mathbf{A}\}$, $d_2 \geq L_2 + \lambda_{\max}\{\mathbf{P} \otimes \mathbf{A}\}$, $\mu_3 = 2^{((1-b_1)/2)+1}/((b_1-1)(Nn)^{((1-b_1)/2)})$, $b_1 > 1$, $\mu_4 = 2^{((1-b_2)/2)+1}/(1-b_2)$, $0 < b_2 < 1$, $\kappa^{b_1}(t) = ((\kappa_1^{b_1}(t))^T, (\kappa_2^{b_1}(t))^T, \dots, (\kappa_N^{b_1}(t))^T)^T$, $\kappa^{b_2}(t) = ((\kappa_1^{b_2}(t))^T, (\kappa_2^{b_2}(t))^T, \dots, (\kappa_N^{b_2}(t))^T)^T$.

Proof We define the nonnegative function as follows:

$$V(t) = \frac{1}{2} \kappa(t)^T \kappa(t).$$

Based on Assumption 1 and the synchronization error neural network (3), one can calculate that

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N \kappa_i^T(t) (F(\kappa_i(t)) + G(\kappa_i(t-\delta(t))) + \sum_{j=1}^N m_{ij} A \kappa_j(t) + \sum_{j=1}^N p_{ij} A \kappa_j(t-\delta(t)) + x_i(t)) \\ &\leq \sum_{i=1}^N L_1 \kappa_i^T(t) \kappa_i(t) + \sum_{i=1}^N L_2 \kappa_i^T(t) \kappa_i(t-\delta(t)) + \sum_{i=1}^N \kappa_i^T(t) (\sum_{j=1}^N m_{ij} A \kappa_j(t) + \sum_{j=1}^N p_{ij} A \kappa_j(t-\delta(t))) \\ &\quad - \sum_{i=1}^N d_1 \kappa_i^T(t) \kappa_i(t) - \sum_{i=1}^N d_2 \kappa_i^T(t) \kappa_i(t-\delta(t)) - \frac{1}{T^\Delta} (\mu_3 \sum_{i=1}^N \kappa_i^T(t) \kappa_i^{b_1}(t) + \mu_4 \sum_{i=1}^N \kappa_i^T(t) \kappa_i^{b_2}(t)) \\ &\leq (L_1 + \lambda_{\max}\{M \otimes A\} - d_1) \kappa^T(t) \kappa(t) + (L_2 + \lambda_{\max}\{P \otimes A\} - d_2) \kappa^T(t) \kappa(t-\delta(t)) - \frac{1}{T^\Delta} (\mu_3 \sum_{i=1}^N \kappa_i^T(t) \kappa_i^{b_1}(t) + \mu_4 \sum_{i=1}^N \kappa_i^T(t) \kappa_i^{b_2}(t)). \end{aligned}$$

As $d_1 \geq L_1 + \lambda_{\max}\{M \otimes A\}$, $d_2 \geq L_2 + \lambda_{\max}\{P \otimes A\}$, one gets

$$\dot{V}(t) \leq -\frac{1}{T^\Delta} (\mu_3 \sum_{i=1}^N \kappa_i^T(t) \kappa_i^{b_1}(t) + \mu_4 \sum_{i=1}^N \kappa_i^T(t) \kappa_i^{b_2}(t)). \quad (7)$$

From Lemma 1, it can easily obtain that

$$\kappa^T(t) \kappa^{b_1}(t) = \sum_{i=1}^N \sum_{j=1}^n (\kappa_{ij}^2(t))^{\frac{1+b_1}{2}} \geq (Nn)^{\frac{1-b_1}{2}} (\kappa^T(t) \kappa(t))^{\frac{1+b_1}{2}} = 2^{\frac{1+b_1}{2}} (Nn)^{\frac{1-b_1}{2}} (V(t))^{\frac{1+b_1}{2}}, \quad (8)$$

and

$$\kappa^T(t) \kappa^{b_2}(t) = \sum_{i=1}^N \sum_{j=1}^n (\kappa_{ij}^2(t))^{\frac{1+b_2}{2}} \geq \sum_{i=1}^N (\sum_{j=1}^n \kappa_{ij}^2(t))^{\frac{1+b_2}{2}} \geq (\sum_{i=1}^N \sum_{j=1}^n \kappa_{ij}^2(t))^{\frac{1+b_2}{2}} = 2^{\frac{1+b_2}{2}} (V(t))^{\frac{1+b_2}{2}}. \quad (9)$$

Using (8) and (9) in (7) leads to

$$\frac{d(V(t))}{dt} \leq -\frac{\mu_3}{T^\Delta} 2^{\frac{1+b_1}{2}} (Nn)^{\frac{1-b_1}{2}} (V(t))^{\frac{1+b_1}{2}} - \frac{\mu_4}{T^\Delta} 2^{\frac{1+b_2}{2}} (V(t))^{\frac{1+b_2}{2}}. \quad (10)$$

When taking $\mu_3 = \frac{2^{\frac{1-b_1}{2}+1}}{(b_1-1)(Nn)^{\frac{1-b_1}{2}}}$ and $\mu_4 = \frac{2^{\frac{1-b_2}{2}+1}}{1-b_2}$, we obtain from (10) that

$$\frac{d(V(t))}{dt} \leq -\frac{1}{T^\Delta} \frac{4}{b_1-1} (V(t))^{\frac{1+b_1}{2}} - \frac{1}{T^\Delta} \frac{4}{1-b_2} (V(t))^{\frac{1+b_2}{2}} = -\frac{1}{T^\Delta} \frac{2}{\frac{b_1+1}{2}-1} (V(t))^{\frac{1+b_1}{2}} - \frac{1}{T^\Delta} \frac{2}{1-\frac{b_2+1}{2}} (V(t))^{\frac{1+b_2}{2}}. \quad (11)$$

If $c_1 = \frac{1+b_1}{2}$ and $c_2 = \frac{1+b_2}{2}$, the following inequality is the direct result of (11),

$$\frac{d(V(t))}{dt} \leq -\frac{1}{T^\Delta} \frac{2}{c_1-1} (V(t))^{c_1} - \frac{1}{T^\Delta} \frac{2}{1-c_2} (V(t))^{c_2} = -\frac{1}{T^\Delta} \mu_1 (V(t))^{c_1} - \frac{1}{T^\Delta} \mu_2 (V(t))^{c_2},$$

where $\mu_1 = \frac{2}{c_1-1}$, $\mu_2 = \frac{2}{1-c_2}$.

So, based on Lemma 3, the synchronization error neural networks (3) is modified FxT stable, and $T \leq T^\Delta$. That is, CNs (1) and (2) are modified FxTS via continuous controller (6). The proof is completed.

Remark 4 Based on the conditions given in Theorem 1, when $x_i(t) = -d_1 \kappa_i(t) - d_2 \kappa_i(t-\delta(t)) - \frac{\mu}{T^\Delta} (\kappa_i^{b_1}(t) + \kappa_i^{b_2}(t))$, $\mu = \max\{\mu_3, \mu_4\}$ (or $\mu = \mu_3 + \mu_4$), CNs (1) and (2) can also achieve modified FxTS, and $T \leq T^\Delta$.

2.2 Modified FxTS with Discontinuous Controller

Theorem 2 Under Assumption 1, the discontinuous controller is given by

$$x_i(t) = -d_1 \kappa_i(t) - d_2 \kappa_i(t-\delta(t)) - \frac{1}{T^\Delta} (\mu_5 \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_1} + \mu_6 \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_2}), \quad (12)$$

then CNs (1) and (2) can realize modified FxTS, and $T \leq T^\Delta$, where $d_1 \geq L_1 + \lambda_{\max}\{M \otimes A\}$, $d_2 \geq L_2 + \lambda_{\max}\{P \otimes A\}$, $\mu_5 = 2^{((1-b_1)/2)+1} / ((b_1-1)(Nn)^{((1-b_1)/2)})$, $b_1 > 1$, $\mu_6 = 2^{((1-b_2)/2)+1} / (1-b_2)$, $0 < b_2 < 1$, $\kappa^{b_1}(t) = ((\kappa_1^{b_1}(t))^T, (\kappa_2^{b_1}(t))^T, \dots, (\kappa_N^{b_1}(t))^T)^T$, $\kappa^{b_2}(t) = ((\kappa_1^{b_2}(t))^T, (\kappa_2^{b_2}(t))^T, \dots, (\kappa_N^{b_2}(t))^T)^T$.

Proof Define a nonnegative function $V(t) = \frac{1}{2} \kappa(t)^T \kappa(t)$.

Similar to Theorem 1, we can find

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N \kappa_i^T(t) (\mathbf{F}(\kappa_i(t)) + \mathbf{G}(\kappa_i(t - \delta(t))) + \sum_{j=1}^N m_{ij} \mathbf{A} \kappa_j(t) + \sum_{j=1}^N p_{ij} \mathbf{A} \kappa_j(t - \delta(t)) + \mathbf{x}_i(t)) \\ &\leq (L_1 + \lambda_{\max} \{\mathbf{M} \otimes \mathbf{A}\} - d_1) \kappa^T(t) \kappa(t) + (L_2 + \lambda_{\max} \{\mathbf{P} \otimes \mathbf{A}\} - d_2) \kappa^T(t) \kappa(t - \delta(t)) \\ &\quad - \frac{1}{T^\Delta} (\mu_5 \sum_{i=1}^N \kappa_i^T(t) \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_1} + \mu_6 \sum_{i=1}^N \kappa_i^T(t) \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_2}). \end{aligned}$$

As $d_1 \geq L_1 + \lambda_{\max} \{\mathbf{M} \otimes \mathbf{A}\}$, $d_2 \geq L_2 + \lambda_{\max} \{\mathbf{P} \otimes \mathbf{A}\}$, and based on Lemma 1, then

$$\begin{aligned} \dot{V}(t) &\leq -\frac{1}{T^\Delta} (\mu_5 \sum_{i=1}^N \kappa_i^T(t) \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_1} + \mu_6 \sum_{i=1}^N \kappa_i^T(t) \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_2}) = -\frac{1}{T^\Delta} (\mu_5 \sum_{i=1}^N \sum_{j=1}^n |\kappa_{ij}(t)|^{1+b_1} + \mu_6 \sum_{i=1}^N \sum_{j=1}^n |\kappa_{ij}(t)|^{1+b_2}) \\ &\leq -\frac{1}{T^\Delta} (\mu_5 (Nn)^{\frac{1-b_1}{2}} (\sum_{i=1}^N \kappa_i^T(t) \kappa_i(t))^{\frac{1+b_1}{2}} + \mu_6 (\sum_{i=1}^N \kappa_i^T(t) \kappa_i(t))^{\frac{1+b_2}{2}}) = -\frac{1}{T^\Delta} (2^{\frac{1+b_1}{2}} \mu_5 (Nn)^{\frac{1-b_1}{2}} (V(t))^{\frac{1+b_1}{2}} + 2^{\frac{1+b_2}{2}} \mu_6 (V(t))^{\frac{1+b_2}{2}}). \end{aligned} \quad (13)$$

Based on (10), it follows from $\mu_5 = \frac{2^{\frac{1-b_1}{2}+1}}{(b_1-1)(Nn)^{\frac{1-b_1}{2}}}$ and $\mu_6 = \frac{2^{\frac{1-b_2}{2}+1}}{1-b_2}$ that

$$\frac{d(V(t))}{dt} \leq -\frac{1}{T^\Delta} \frac{4}{b_1-1} (V(t))^{\frac{1+b_1}{2}} - \frac{1}{T^\Delta} \frac{4}{1-b_2} (V(t))^{\frac{1+b_2}{2}}. \quad (14)$$

The following proof is identical to Theorem 1, so it is omitted here. As a result, CNs (1) and (2) are modified FxTS via discontinuous controller (12). The proof is completed.

Remark 5 On the basis of the conditions given in Theorem 2, when $\mathbf{x}_i(t) = -d_1 \kappa_i(t) - d_2 \kappa_i(t - \delta(t)) - \frac{\mu}{T^\Delta} (\text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_1} + \text{sign}(\kappa_i(t)) |\kappa_i(t)|^{b_2})$, $\mu = \max\{\mu_5, \mu_6\}$ (or $\mu = \mu_5 + \mu_6$), CNs (1) and (2) are modified FxTS, and $T \leq T^\Delta$.

Remark 6 In this paper, the controller (6) and (12) contain linear terms $-d_1 \kappa_i(t)$ and $-d_2 \kappa_i(t - \delta(t))$, and based on $d_1 \geq L_1 + \lambda_{\max} \{\mathbf{M} \otimes \mathbf{A}\}$, $d_2 \geq L_2 + \lambda_{\max} \{\mathbf{P} \otimes \mathbf{A}\}$ and Lemma 3, CNs (1) and (2) can realize modified FxTS at ST T , before any given time T^Δ . Although the value of d_1 and d_2 are related to L_1 , L_2 , $\mathbf{M}$, $\mathbf{P}$, $\mathbf{A}$, the ST obtained is unrelated to the initial values and disturbed parameters of error neural networks (3), so it is more useful compared with the existing results^[24, 28].

3 A Simulation Example

At last, an appropriate simulation example is given to confirm the effectiveness and feasibility of the strategies of continuous and discontinuous control.

We consider the following CNs with three identical nodes:

$$\dot{\mathbf{r}}_i(t) = \mathbf{f}(\mathbf{r}_i(t)) + \mathbf{g}(\mathbf{r}_i(t - \delta(t))) + \sum_{j=1}^3 m_{ij} \mathbf{A} \mathbf{r}_j(t) + \sum_{j=1}^3 p_{ij} \mathbf{A} \mathbf{r}_j(t - \delta(t)), \quad (15)$$

where $i = 1, 2, 3$, $\delta(t) = \cos(t)$,

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}, \mathbf{M} = \begin{pmatrix} -0.6 & 0.2 & 0.4 \\ 0.2 & -0.4 & 0.2 \\ 0.3 & 0 & -0.3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.1 & 0 \\ 0.2 & 0 & -0.2 \end{pmatrix}, \\ \mathbf{f}(\mathbf{r}_i(t)) &= [\mathbf{f}_1(\mathbf{r}_{i1}(t)), \mathbf{f}_2(\mathbf{r}_{i2}(t))]^T, \mathbf{f}_i(\mathbf{r}) = -\frac{1}{2} (|\mathbf{r} + 1| - |\mathbf{r} - 1|), \\ \mathbf{g}(\mathbf{r}_i(t - \delta(t))) &= [\mathbf{g}_1(\mathbf{r}_{i1}(t - \delta(t))), \mathbf{g}_2(\mathbf{r}_{i2}(t - \delta(t)))]^T, \mathbf{g}_i(\mathbf{r}) = \frac{1}{10} \tanh(\mathbf{r}). \end{aligned}$$

The controlled CNs corresponding to (15) can be described as:

$$\dot{\mathbf{w}}_i(t) = \mathbf{f}(\mathbf{w}_i(t)) + \mathbf{g}(\mathbf{w}_i(t - \delta(t))) + \sum_{j=1}^3 m_{ij} \mathbf{A} \mathbf{w}_j(t) + \sum_{j=1}^3 p_{ij} \mathbf{A} \mathbf{w}_j(t - \delta(t)) + \mathbf{x}_i(t), \quad (16)$$

where $i = 1, 2, 3$, $\mathbf{x}_i(t)$ is the controller.

Consequently, the error dynamical networks are given by:

$$\begin{aligned} \dot{\kappa}_i(t) = & \mathbf{F}(\kappa_i(t)) + \mathbf{G}(\kappa_i(t - \delta(t))) + \sum_{j=1}^3 m_{ij} \mathbf{A} \kappa_j(t) \\ & + \sum_{j=1}^3 p_{ij} \mathbf{A} \kappa_j(t - \delta(t)) + \mathbf{x}_i(t), \end{aligned} \quad (17)$$

where

$$\mathbf{F}(\kappa_i(t)) = \mathbf{f}(\mathbf{w}_i(t)) - \mathbf{f}(\mathbf{r}_i(t)),$$

$$\mathbf{G}(\kappa_i(t - \delta(t))) = \mathbf{g}(\mathbf{w}_i(t - \delta(t))) - \mathbf{g}(\mathbf{r}_i(t - \delta(t))).$$

From Assumption 1, we can obtain that $L_1 = 1$, $L_2 = 0.1$. Let $b_1 = 1.5$, $b_2 = 0.7$, $\mu_3 = \mu_5 = 8.2391$, $\mu_4 = \mu_6 = 7.3971$, $d_1 = 4$, $d_2 = 1.1$, it is easy to see that these conditions satisfy Theorem 1 and Theorem 2.

Next, this study prove the validity of the control strategies. The initial values of CNs (15) for simulation are chosen as $\mathbf{r}_1(t) = [0.3, -0.1]^T$, $\mathbf{r}_2(t) = [0.2, -0.2]^T$, $\mathbf{r}_3(t) = [0.5, -0.2]^T$, $t \in [-1, 0]$.

The initial values of CNs (16) are chosen as

$$\mathbf{w}_1(t) = [0.7, -0.7]^T, \mathbf{w}_2(t) = [0.4, -0.3]^T,$$

$$\mathbf{w}_3(t) = [0.6, -0.6]^T, t \in [-1, 0].$$

Based on the initial values of CNs (15) and (16) given above, the simulation results are presented in Figs. 1-3. Figure 1 simulates the evolution of error dynamical networks (17) without controller. It shows error dynamical networks (17) cannot converge to zero which means CNs (15) and (16) are not synchronized. When $T^* = 0.1$, Figure 2 gives the evolution of error dynamical networks (17) under continuous controller (6), and the ST T is 0.06. The evolution of error dynamical networks (17) under discontinuous controller (12) is given in Fig. 3, and the ST T is 0.03. From Figs. 2 and 3, it is not difficult to see that the CNs (15) and (16) can achieve synchronization. This shows that the control strategies proposed in this paper are effective.

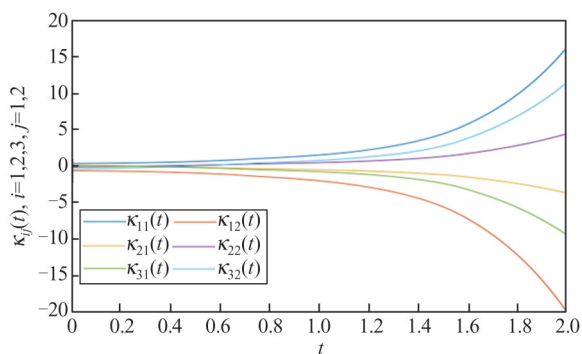


Fig. 1 The error networks (17) without controller

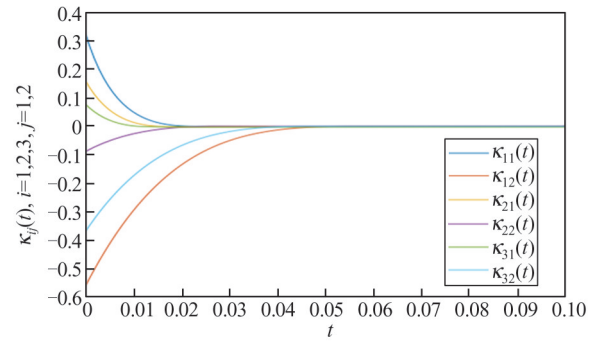


Fig. 2 The error networks (17) with controller (6)

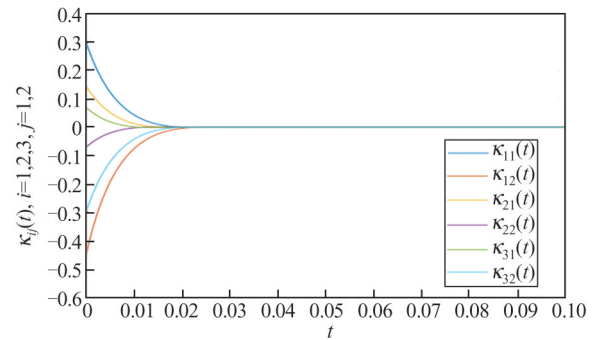


Fig. 3 The error networks (17) with controller (12)

4 Conclusion

In this paper, based on continuous and discontinuous control strategies, modified FxTS criteria of CNs with time-varying delays have been addressed, where the ST is determined by an arbitrary positive number given in advance, so the ST is not correlated with either the initial value or the parameters of the CNs. An appropriate simulation example is provided to show the effectiveness of the strategies of continuous and discontinuous control. Further investigation may aim to design the controller for modified FxTS of impulsive CNs.

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基于连续或不连续控制的时变时滞复杂网络的修正固定时间同步判据

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摘要: 本文基于连续和不连续控制器, 研究了带有时变时滞的复杂网络的修正固定时间同步。首先, 为了保证固定时间同步的确定时间与复杂网络的初始值和参数都无关, 提出了一个修正的固定时间稳定性定理, 其中确定时间是由提前给定的任意正数决定。然后, 设计了连续控制器和不连续控制器, 用于实现复杂网络的固定时间同步目标。基于所设计的控制器, 复杂网络可以在任意给定时间甚至更早实现同步。并且控制策略有效解决了确定时间与复杂网络参数有关的问题。最后通过仿真实例验证了所设计控制策略的有效性。

关键词: 复杂网络; 确定时间; 固定时间同步; 控制器; 时变时滞

□