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Blow-Up for a Pseudo-Parabolic Equation with Memory and Convection Terms

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Abstract: In this article, we deal with the blow-up phenomenon of a pseudo-parabolic equation with memory and convection terms. By constructing some appropriate auxiliary functions and using the modified concavity method, the blow-up criterion of the weak solution is given.

Key words: blow-up; pseudo-parabolic equation; memory term; convection term

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0 Introduction

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with sufficiently smooth boundary $\partial\Omega$. The main objective of this article is to investigate the blow-up phenomenon of the following pseudo-parabolic equation with memory and convection terms

$$\begin{cases} \left(\Delta u - |u|^p u\right)_t + \Delta u + u^q u_{x_1} + |u|^m u = \int_0^t \mathrm{e}^{-(t-s)}\Delta u \mathrm{d}s, & (\boldsymbol{x}, t) \in \Omega_T, \\ u(\boldsymbol{x}, t) = 0, & (\boldsymbol{x}, t) \in S_T, \\ u(\boldsymbol{x}, 0) = u_0(\boldsymbol{x}), & \boldsymbol{x} \in \overline{\Omega}, \end{cases} \tag{1}$$

where m, p, q and T are four positive parameters, $\boldsymbol{x} = (x_1, x_2, \cdots, x_N) \in \Omega$, $\Omega_T = \Omega \times (0, T)$, $S_T = \partial\Omega \times (0, T)$, and $u_0(\boldsymbol{x}) \in L^\infty(\Omega) \cap H_0^1(\Omega)$.

The pseudo-parabolic equations are characterized by the occurrence of mixed third-order derivatives, more precisely, second in space variable and first order in time variable. Such equations are used to model heat conduction in two-temperature system[1-2], fluid flow in porous medium[3], two phase flow in porous medium with dynamical capillary pressure[4], and the populations with the tendency to form crows[5-6]. The pseudo-parabolic equations with memory term, like the form (1), can be used to describe the non-stationary process of the electric potential in semiconductors with sources of free-charge currents[7-8]. In this physics content, $u(\boldsymbol{x}, t)$ stands for the electric field potential, the memory term $\int_0^t \mathrm{e}^{-(t-s)}\Delta u \mathrm{d}s$ stands for the memory effect, $u^q u_{x_1}$ stands for the nonlinear convection of the free electrons, and $|u|^m u$

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stands for a nonlinear source of the free electric current.

In the past period of time, many mathematicians have focused their attention on the properties of solutions to various pseudo-parabolic equations[9-12]. In particular, Meyvaci[13] considered the problem

$$\begin{cases} u_t-\Delta u_t-\Delta u-u^p u_{x_1}=|u|^{2m}u, \quad (\boldsymbol{x},t)\in\Omega_T, \\ u(\boldsymbol{x},t)=0, \quad (\boldsymbol{x},t)\in S_T, \\ u(\boldsymbol{x},0)=u_0(\boldsymbol{x}), \quad \boldsymbol{x}\in\overline{\Omega}, \end{cases} \tag{2}$$

and proved that the solution of (2) with $p\in(1,m]$ and large initial value blows up in finite time. Korpusov and Sveshnikov[14] dealt with the blow-up behavior of the following pseudo-parabolic equation with non-local term

$$\begin{cases} \left(\Delta u-u-|u|^p u\right)_t+\Delta u+u_{x_1}+uu_{x_1}=\Delta u\int_\Omega|\nabla u|^2\mathrm{d}x, \quad (\boldsymbol{x},t)\in\Omega_T, \\ u(\boldsymbol{x},t)=0, \quad (\boldsymbol{x},t)\in S_T, \\ u(\boldsymbol{x},0)=u_0(\boldsymbol{x}), \quad \boldsymbol{x}\in\overline{\Omega}. \end{cases} \tag{3}$$

Under some certain conditions, by constructing appropriate auxiliary functions and using differential inequality techniques, the authors gave the sufficient condition for the occurrence of the blow-up phenomenon. Ptashnyk[15] studied the degenerate quasi-linear pseudo-parabolic equation with memory term and variational inequality as the form

$$\begin{cases} \partial_t b^j(u)-\nabla\cdot\left(a(\boldsymbol{x})\nabla u_t\right)^j-\nabla\cdot d^j(t,\boldsymbol{x},u,\nabla u)+M^j(u)=f^j(u), \quad (\boldsymbol{x},t)\in\Omega_T, \\ u^j(\boldsymbol{x},t)=0, \quad (\boldsymbol{x},t)\in S_T, \\ b^j\left(u(\boldsymbol{x},0)\right)=b^j\left(u_0(\boldsymbol{x})\right), \quad \boldsymbol{x}\in\overline{\Omega}, \end{cases} \tag{4}$$

where the memory operator $M^j(u)$ is defined by

$$\left(M^j(t)(u),v^j\right)=\int_\Omega\int_0^t K^j(t,s)\,g^j\left(s,\boldsymbol{x},\nabla u(s,\boldsymbol{x})\right)\mathrm{d}s\nabla v^j(t,\boldsymbol{x})\,\mathrm{d}x$$

for all functions $u,v\in L^p\left(0,T;H_0^{1,p}(\Omega)^l\right)$ with $p\geqslant 2$, and $(\cdot,\cdot)$ means the inner product in the sense of $L^2(\Omega)$. Under some suitable assumptions on the vector field b, namely, $b\colon\mathbb{R}^l\to\mathbb{R}^l$ is monotone non-decreasing and a continuous gradient. Ptashnyk[15] obtained the local existence result of the weak solution to problem (4) by using Rothe-Galerkins method. Moreover, he also got the uniqueness of the weak solution for some special cases.

To the best of our knowledge, there is no relevant literature on the blow-up property of the solution to problem (1). Inspired by the works mentioned above, the main goal of this article is to give a criterion for finite time blow-up phenomenon.

1 Preliminaries and Main Result

Throughout this article, we work with the weak solution of problem (1) in the sense of the following definition.

Definition 1 A function $u(\boldsymbol{x},t)$ with $u(\boldsymbol{x},0)=u_0(\boldsymbol{x})$ a.e. $\boldsymbol{x}\in\overline{\Omega}$ is said to be a weak solution of problem (1) if $u\in L^2\left(0,T;H_0^1(\Omega)\right)\cap L^\infty\left(0,T;H_0^1(\Omega)\right)$, $|u|^p u\in L^\infty\left(0,T;L^1(\Omega)\right)$, $\left(|u|^p u-\Delta u\right)_t\in L^2\left(0,T;H^{-1}(\Omega)\right)$, and for any $v\in L^2\left(0,T;H_0^1(\Omega)\right)$ with $v_t\in L^2\left(0,T;H_0^1(\Omega)\right)\cap L^1\left(0,T;L^\infty(\Omega)\right)$ and $v(\boldsymbol{x},T)=0$, one has

$$\begin{aligned} &\int_0^T\int_\Omega\left(|u|^p uv_t+\nabla u\cdot\nabla v_t\right)\mathrm{d}x\mathrm{d}t-\int_0^T\int_\Omega\left(|u_0|^p u_0v_t+\nabla u_0\cdot\nabla v_t\right)\mathrm{d}x\mathrm{d}t+\int_0^T\int_\Omega\int_0^t \mathrm{e}^{-(t-s)}\nabla u(\boldsymbol{x},s)\cdot\nabla v(\boldsymbol{x},t)\,\mathrm{d}s\mathrm{d}x\mathrm{d}t \\ &=\int_0^T\int_\Omega\left(\nabla u\cdot\nabla v+\frac{1}{q+1}u^{q+1}v_{x_1}-|u|^m uv\right)\mathrm{d}x\mathrm{d}t. \end{aligned} \tag{5}$$

Following a slight modification of the proof of Theorem 3 in Ref. [15], we can obtain the local existence result of the weak solution for problem (1). The main result of this article is the following theorem.

Theorem 1 Suppose that $0<\max\{p,2q\}<m$, and the initial data $u_0(x)$ satisfies

$$\|u_0\|_{m+2}^{m+2}>\delta_1\|\nabla u_0\|_2^2+\delta_2\|u_0\|_{p+2}^{p+2}+\delta_3.$$

Then

$$\lim_{t\to T^-}\left(\frac{1}{2}\|\nabla u\|_2^2+\frac{p+1}{p+2}\|u\|_{p+2}^{p+2}\right)=+\infty,$$

where T^- is given by (31), and

$$\delta_1=1+\frac{1}{2}\left(\frac{\beta}{\alpha-1}+\sqrt{\frac{\gamma}{\alpha-1}}\right),\delta_2=\frac{p+1}{p+2}\left(\frac{\beta}{\alpha-1}+\sqrt{\frac{\gamma}{\alpha-1}}\right),\delta_3=C\left(\frac{\beta}{\alpha-1}+\sqrt{\frac{\gamma}{\alpha-1}}\right),$$

$\alpha>1$, β, γ and C are positive constants, given in Section 2.

2 Proof of the Main Result

In this section, by constructing some appropriate auxiliary functions and using the modified concavity method, we are going to give the proof of Theorem 1.

Proof First, we denote

$$G(t)=\frac{1}{2}\|\nabla u\|_2^2+\frac{p+1}{p+2}\|u\|_{p+2}^{p+2}+C, \tag{6}$$

and

$$H(t)=\|\nabla u_t\|_2^2+(p+1)\int_\Omega|u|^p|u_t|^2\mathrm{d}x, \tag{7}$$

where $C>0$ is to be determined. Multiplying both sides of the first equation in (1) by u and u_t, respectively, and then integrating over Ω, we have

$$G'(t)=\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\mathrm{d}x\mathrm{d}s-\|\nabla u\|_2^2+\|u\|_{m+2}^{m+2}, \tag{8}$$

and

$$H(t)=\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u_t(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\mathrm{d}x\mathrm{d}s+\frac{1}{m+2}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|_{m+2}^{m+2}-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2-\frac{1}{q+1}\int_\Omega u^{q+1}u_{tx_1}\mathrm{d}x. \tag{9}$$

Taking the derivative of $G(t)$ with respect to t, we get

$$G'(t)=\int_\Omega\nabla u\cdot\nabla u_t\mathrm{d}x+(p+1)\int_\Omega u^{p+1}u_t\mathrm{d}x. \tag{10}$$

Applying the Cauchy-Schwarz inequality, we can conclude that

$$\begin{aligned}\left[G'(t)\right]^2&=\left[(\nabla u,\nabla u_t)+\left(\sqrt{p+1}\,u^{\frac{p}{2}+1},\sqrt{p+1}\,u^{\frac{p}{2}}u_t\right)\right]^2\\&\leqslant\left(\|\nabla u\|_2^2+(p+1)\|u\|_{p+2}^{p+2}\right)\left(\|\nabla u_t\|_2^2+(p+1)\int_\Omega|u|^p|u_t|^2\mathrm{d}x\right)\leqslant(p+2)G(t)H(t),\end{aligned} \tag{11}$$

where $(\cdot,\cdot)$ is the inner product in the sense of $L^2(\Omega)$. On the other hand, by virtue of (8) and (9), it is obvious that

$$\begin{aligned}H(t)=&\frac{1}{m+2}G''(t)+\frac{m+1}{m+2}\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u_t(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\mathrm{d}x\mathrm{d}s\\&+\frac{1}{m+2}\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\mathrm{d}x\mathrm{d}s\\&-\frac{m}{2(m+2)}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2-\frac{1}{q+1}\int_\Omega u^{q+1}u_{tx_1}\mathrm{d}x-\frac{1}{m+2}\|\nabla u\|_2^2.\end{aligned} \tag{12}$$

Making use of the Cauchy-Schwarz inequality again, we have

$$\left|\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla u\|_2^2\right|\leqslant\frac{\varepsilon_0}{2}\|\nabla u_t\|_2^2+\frac{1}{2\varepsilon_0}\|\nabla u\|_2^2, \tag{13}$$

and

$$\int_\Omega|u|^{q+1}\left|u_{tx_1}\right|\mathrm{d}x\leqslant\frac{\varepsilon_1}{2}\|\nabla u_t\|_2^2+\frac{1}{2\varepsilon_1}\|u\|_{2q+2}^{2q+2}, \tag{14}$$

where ε_0 and ε_1 are two positive constants, which will be given later. Moreover, we have

$$\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u_t(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\,\mathrm{d}x\mathrm{d}s \leqslant \frac{\varepsilon_2}{2}(1-\mathrm{e}^{-t})\|\nabla u_t\|_2^2+\frac{1}{2\varepsilon_2}\int_0^t \mathrm{e}^{-(t-s)}\|\nabla u(\boldsymbol{x},s)\|_2^2\mathrm{d}s$$
$$\leqslant \frac{\varepsilon_2}{2}\|\nabla u_t\|_2^2+\frac{1}{\varepsilon_2}\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s, \tag{15}$$

here $\varepsilon_2>0$ will be chosen later. Likewise, we can obtain that

$$\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\,\mathrm{d}x\mathrm{d}s \leqslant \frac{1}{2}\|\nabla u\|_2^2+\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s. \tag{16}$$

Combining (12)-(16), we arrive at

$$H(t)\leqslant \frac{1}{m+2}G''(t)+\left[\frac{m\varepsilon_0}{4(m+2)}+\frac{\varepsilon_1}{2(q+1)}+\frac{\varepsilon_2(m+1)}{2(m+2)}\right]\|\nabla u_t\|_2^2$$
$$+\left[\frac{3}{2(m+2)}+\frac{m}{4\varepsilon_0(m+2)}\right]\|\nabla u\|_2^2+\frac{1}{2\varepsilon_1(q+1)}\|u\|_{2q+2}^{2q+2}+\left[\frac{m+1}{\varepsilon_2(m+2)}+\frac{1}{m+2}\right]\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s. \tag{17}$$

Noticing that $m>2q$, Young's inequality can be used to obtain

$$\|u\|_{2q+2}^{2q+2}\leqslant \frac{m-2q}{m+2}|\Omega|+\frac{2q+2}{m+2}\|u\|_{m+2}^{m+2}. \tag{18}$$

On the other hand, using (8) and (16), we are easy to find that

$$\|u\|_{m+2}^{m+2}=G'(t)+\|\nabla u\|_2^2-\int_0^t\int_\Omega \mathrm{e}^{-(t-s)}\nabla u(\boldsymbol{x},t)\cdot\nabla u(\boldsymbol{x},s)\,\mathrm{d}x\mathrm{d}s$$
$$\leqslant G'(t)+\frac{3}{2}\|\nabla u\|_2^2+\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s. \tag{19}$$

From (17), (18) and (19), it follows that

$$H(t)\leqslant \frac{1}{m+2}G''(t)+\frac{1}{\varepsilon_1(m+2)}G'(t)+\left[\frac{3}{2(m+2)}+\frac{m}{4\varepsilon_0(m+2)}+\frac{3}{2\varepsilon_1(m+2)}\right]\|\nabla u\|_2^2$$
$$+\left[\frac{m\varepsilon_0}{4(m+2)}+\frac{\varepsilon_1}{2(q+1)}+\frac{\varepsilon_2(m+1)}{2(m+2)}\right]\|\nabla u_t\|_2^2$$
$$+\left[\frac{1}{m+2}+\frac{1}{\varepsilon_1(m+2)}+\frac{m+1}{\varepsilon_2(m+2)}\right]\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s+\frac{|\Omega|(m-2q)}{2\varepsilon_1(m+2)(q+1)}. \tag{20}$$

Furthermore, the term $\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s$ can be estimated as follows:

$$\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s=\mathrm{e}^{-t}\int_0^t \mathrm{e}^{s}G(s)\,\mathrm{d}s=G(t)-\mathrm{e}^{-t}G(0)-\int_0^t \mathrm{e}^{-(t-s)}G'(s)\,\mathrm{d}s\leqslant G(t)-\int_0^t \mathrm{e}^{-(t-s)}G'(s)\,\mathrm{d}s. \tag{21}$$

Now, from the expressions of the constants δ_i, $i=1, 2, 3$, it follows that $\delta_1>1$, $\delta_2>0$ and $\delta_3>0$. Then by the assumption $\|u_0\|_{m+2}^{m+2}>\delta_1\|\nabla u_0\|_2^2+\delta_2\|u_0\|_{p+2}^{p+2}+\delta_3$, we have $\|u_0\|_{m+2}^{m+2}>\|\nabla u_0\|_2^2$. This tells us that $G'(0)=\|u_0\|_{m+2}^{m+2}-\|\nabla u_0\|_2^2>0$. Hence, there exists a $t_1\in(0,+\infty)$ such that $G'(t)\geqslant 0$ holds for any $t\in[0,t_1]$. Collecting this fact and (21), we have

$$\int_0^t \mathrm{e}^{-(t-s)}G(s)\,\mathrm{d}s\leqslant G(t),\ \ t\in[0,\ t_1]. \tag{22}$$

From (6) and (7), it follows that

$$\|\nabla u_t\|_2^2\leqslant H(t), \tag{23}$$

and

$$\|\nabla u\|_2^2\leqslant 2G(t)-2C. \tag{24}$$

In light of the constraint that $p\in(0,m)$, one can choose suitable ε_0, ε_1 and ε_2 such that

$$C_1=\frac{m\varepsilon_0}{4(m+2)}+\frac{\varepsilon_1}{2(q+1)}+\frac{\varepsilon_2(m+1)}{2(m+2)}\in\left(0,\frac{m-p}{m+2}\right)\subset(0,1).$$

For the fixed ε_0, ε_1 and ε_2 above, taking

$$C=\frac{\varepsilon_0|\Omega|(m-2q)}{(q+1)(6\varepsilon_0+m\varepsilon_1+6\varepsilon_0\varepsilon_1)},$$

then (20)-(24) tell us that

$$(m+2)(1-C_1)H(t)\leqslant G''(t)+\frac{1}{\varepsilon_1}G'(t)+\left(4+\frac{m}{2\varepsilon_0}+\frac{4}{\varepsilon_1}+\frac{m+1}{\varepsilon_2}\right)G(t). \tag{25}$$

Inserting (25) in (11), we have

$$G(t)G''(t)-\alpha\left(G'(t)\right)^2+\beta G(t)G'(t)+\gamma G^2(t)\geqslant 0,\ \ t\in[0,t_1], \tag{26}$$

where

$$\alpha=\frac{(m+2)(1-C_1)}{P+2}>1,\beta=\frac{1}{\varepsilon_1},\gamma=4+\frac{m}{2\varepsilon_0}+\frac{4}{\varepsilon_1}+\frac{m+1}{\varepsilon_2}.$$

Now, setting $Z(t)=G^{1-\alpha}(t)\mathrm{e}^{\beta t}$, then (26) tells us that

$$Z''(t)-\beta Z'(t)-\gamma(\alpha-1)Z(t)\leqslant 0,\ \ t\in[0,t_1]. \tag{27}$$

In light of the constraint that the assumption $\|u_0\|_{m+2}^{m+2}>\delta_1\|\nabla u_0\|_2^2+\delta_2\|u_0\|_{p+2}^{p+2}+\delta_3$, we have

$$G'(0)>\left(\frac{\beta}{\alpha-1}+\sqrt{\frac{\gamma}{\alpha-1}}\right)G(0)>\frac{\beta}{\alpha-1}G(0). \tag{28}$$

Then there exists a $t_2\in(0,t_1]$ such that, for any $t\in[0,t_2]$, $Z'(t)=(\alpha-1)\mathrm{e}^{\beta t}G^{-\alpha}(t)\left(\frac{\beta}{\alpha-1}G(t)-G'(t)\right)\leqslant 0$. The above inequality together with (27), leads to $Z''(t)-\gamma(\alpha-1)Z(t)\leqslant 0,\ \ t\in[0,\ t_2]$. Multiplying both sides of the above inequality by $Z'(t)$ results in $Z'(t)Z''(t)-\gamma(\alpha-1)Z(t)Z'(t)\geqslant 0,\ \ t\in[0,t_2]$. Namely,

$$\left[\left(Z'(t)\right)^2-\gamma(\alpha-1)Z^2(t)\right]'\geqslant 0,\ \ t\in[0,t_2],$$

which implies that

$$\begin{aligned}\left(Z'(t)\right)^2&\geqslant\left[Z'(0)\right]^2+\gamma(\alpha-1)\left[Z^2(t)-Z^2(0)\right]\\&\geqslant(\alpha-1)^2G^{-2\alpha}(0)\left\{\left[G'(0)-\frac{\beta}{\alpha-1}G(0)\right]^2-\frac{\gamma}{\alpha-1}G^2(0)\right\},\ \ t\in[0,t_2].\end{aligned} \tag{29}$$

Combining (28) with (29) yields that

$$Z'(t)\leqslant-(\alpha-1)G^{-\alpha}(0)\sqrt{\left[G'(0)-\frac{\beta}{\alpha-1}G(0)\right]^2-\frac{\gamma}{\alpha-1}G^2(0)}=B. \tag{30}$$

Integrating the above differential inequality from 0 to t, we have $Z(t)\leqslant Z(0)-Bt$, which implies that $G(t)\geqslant\frac{\mathrm{e}^{\frac{\beta}{\alpha-1}t}}{\sqrt[\alpha-1]{G^{1-\alpha}(0)-Bt}}$. That is, $G(t)$ will tend to ∞ as

$$t\to T^-=\frac{1}{BG^{\alpha-1}(0)}=\frac{1}{B\left(\frac{1}{2}\|\nabla u_0\|_2^2+\frac{p+1}{p+2}\|u_0\|_{p+2}^{p+2}+C\right)^{\alpha-1}}<\infty. \tag{31}$$

This completes the proof of Theorem 1.

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具记忆和对流项的伪抛物方程解的爆破

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摘要：通过构造合适的辅助函数，结合改进的凹方法，本文给出了一类具记忆和对流项的伪抛物方程解的爆破准则。

关键词：爆破；伪抛物方程；记忆项；对流项

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